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Markov analysis of the PRMA protocol for local wireless networks *

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PRMA (packet reservation multiple access) is a reservation-ALOHA access protocol specifically designed for wireless microcellular networks that handle both real-time and non-real-time traffic. We present a thorough analysis of this protocol, considering real-time traffic only, based on a suitable Markov model. The size of the model is such that it can be directly used for an exact quantitative analysis of the system. In particular, we are able to analyze the packet dropping process, by evaluating both average and distribution measures. The latter are particularly useful to characterize the degradation caused to real-time traffic (e.g., voice) by the loss of consecutive packets. Besides, we also derive from the Markov model a qualitative analysis of the system stability, based on the equilibrium point analysis (EPA) technique. By this technique, we characterize the system stability and analyze the effect on it of several system parameters (e.g., load, permission probability).

1. Introduction

The growing availability of mobile tools for personal computing and communication is stimulating an intense interest in wireless communication networks. Among the topics of interest there is the definition of access protocols that can efficiently handle both real-time (e.g., voice) and non-real-time (e.g., data) traffic. These protocols must be designed taking into account the expectation of large mobile users densities in the near future, and the limitation in the available radio spectrum. Microcellular networks are a possible solution, thanks to a higher frequency reuse but, in turn, they imply an increasing complexity of mobility management. Hence, access protocols within each microcell should require little or no central coordination, to free up network resources for mobility management [3].

The packet reservation multiple access (PRMA) protocol has been recently proposed as a viable solution for wireless microcellular networks [3]. PRMA is basically a modification of the R-ALOHA protocol [2] for microcellular applications designed for transmission of voice and data. As R-ALOHA, PRMA shares both the advantages of decentralized packet contention protocols and of reservation protocols, better suited for real-time traffic [3].

In a PRMA system, voice terminals are statistically multiplexed to achieve efficient use of the channel resource. To this end, terminals employ speech activity detectors and transmit only during voice active periods (talkspurts). The resulting contending mechanism causes packet delay and, since speech packets require prompt delivery, voice terminals are designed to drop those delayed beyond the maximum delay limit. Dropped packets affect speech quality and hence packet dropping measures are important for assessing the PRMA system performance and the achievable statistical multiplexing gain.

The PRMA voice protocol was first analyzed by simulation in [3]. Successively, in [8,9], Markov models for the PRMA voice and voice-data system were developed. In both papers, though, the system performance was evaluated only approximately by means of equilibrium point analysis (EPA) [11,12], owing to the models complexity. Models that allow direct and exact analysis have also been considered. In [13], the voice-data system is studied under the assumptions the voice terminals do not drop packets. In [6], the PRMA voice system is studied and compared with other random access protocols. The packet dropping probability and the packet loss distribution are evaluated under the assumption that the maximum packet delay is equal to the frame duration.

In this paper we analyze the performance of a PRMA system with emphasis on real-time, delay constrained traffic. We develop a Markov model of the PRMA voice system, the size of which allows direct evaluation of the system performance. We also derive from the Markov model a qualitative analysis of the system stability, based on EPA. The study of the dynamics of a single terminal enables us to analyze the packet dropping process, by evaluating both average and distribution measures. The latter are particularly useful to characterize the degradation caused to real-time traffic (e.g., voice) by the loss of consecutive packets [1]. Differently from [6], in our analysis the maximum packet delay is not constrained to the frame duration. Therefore, our model can be used to study the impact of different delay constraints on system performance.

The rest of the paper is organized as follows. In section 2 we briefly describe the PRMA protocol. In section 3 we present the PRMA model: first, the voice model is described; then, the overall PRMA model is presented. In section 4 we evaluate the packet loss performance measures

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of interest. Stability analysis is addressed in section 5 by means of EPA analysis and the concept of PRMA load line. In section 6 we present and discuss numerical examples as an application of obtained results. Section 7 concludes the paper.

2. The packet reservation multiple access protocol

In this section we briefly describe the PRMA protocol. We concentrate on the real-time (voice) system aspects only. A detailed description of the protocol can be found in [4,9].

PRMA is a contention-based channel access protocol proposed for wireless communications networks. Terminals transmit packetized information over a shared channel to a base station. As R-ALOHA, the PRMA channel is slotted; slots are grouped together into frames. The slot duration is equal to the packet transmission time.

Voice terminals produce a pattern of talkspurts (active) and silent periods. By using a speech detector, only talkspurts are packetized for transmission. During a talkspurt, a new packet is generated every frame and stored in a finite FIFO buffer. No packets are generated during silent periods.

As soon as the first packet of a talkspurt is generated, a terminal begins to contend with other terminals for unreserved slots. As in the R-ALOHA protocol, a contending terminal transmits a packet in a available slot if it has "permission". Permission occurs with a fixed probability at each unreserved slot, independently at each terminal. At the end of each slot, the base broadcasts a feedback packet with the result of the transmission. Due to the limited size of a microcell, the propagation delay is negligible. This allows each station to immediately know the result of a transmission attempt. If two or more contending terminals has attempted to transmit a packet in the same unreserved slot, a collision occurs; the base is unable to detect any packet and the terminals have to retransmit. Instead, if only one terminal has attempted to transmit in an unreserved slot, the base successfully receives the packet and grants the terminal a "reservation" for that slot, i.e., exclusive use of that slot in future frames until it has no more packets to transmit. At the end of the talkspurt, the terminal releases the reservation by leaving the slot empty.

Because of the stringent time constraint of voice applications (typically tens to few hundreds of milliseconds), a terminal drops voice packets that have been delayed beyond the maximum holding time $D_{\max}$. Moreover, in the case that a talkspurt ends before a reservation has been obtained, a terminal drops all the remaining packets in the buffer. Under the assumption that the oldest packet is discarded when a new packet arrives at a full buffer, the maximum holding time $D_{\max}$ is enforced by choosing a buffer size

$$B = \left\lceil \frac{D_{\max}}{N} \right\rceil, \tag{1}$$

where $D_{\max}$ is measured in slot, N denotes the number of slots per frame and $\lceil x \rceil$ denotes the smallest integer larger or equal to x.

3. The model

In this section we develop a Markov model for a PRMA voice system. We begin our study by describing the voice source model.

3.1. The voice model

During a speech, talkspurt and silent periods alternate. A simple model for voice sources is provided by a two state Markov process: exponentially distributed talking (active) periods alternate with exponentially distributed silent (idle) periods. For the analysis of slotted systems, a discrete time version of the above process is preferable. Denote t_1 and t_2 the mean length of a talking and of a silence period, respectively, and τ the slot duration. The probability γ (σ) that a talkspurt (silent) period of mean t_1 (t_2) ends within a slot of duration τ is

$$\gamma = 1 - \exp(-r/t_1) \quad \text{and} \quad \sigma = 1 - \exp(-r/t_2).$$

A discrete time model for a voice source is given in figure 1, where the time unit corresponds to one slot duration. The transition from the *Tal* (talking) to the *Sil* (silent) state occurs with a fixed probability γ, and the transition from the silent to the talking state occurs with a fixed probability σ. The silent and the talking periods are geometrically distributed with means $1/\sigma$ and $1/\gamma$, respectively. The fraction of time spent in each state is

$$\pi_{\text{Sil}} = \frac{\gamma}{\sigma + \gamma} \quad \text{and} \quad \pi_{\text{Talk}} = \frac{\sigma}{\sigma + \gamma},$$

respectively.

Observe that because PRMA voice packets are generated at frame rate, a talkspurt of length L slots in the model of figure 1 corresponds to the generation of $L_p = \lceil L/N \rceil$ packets, where N denotes the number of slots per frame.

3.2. The PRMA model

Consider a PRMA system with M homogeneous independent voice terminals. Let N denote the number of slots per frame and p the permission probability, that we assume

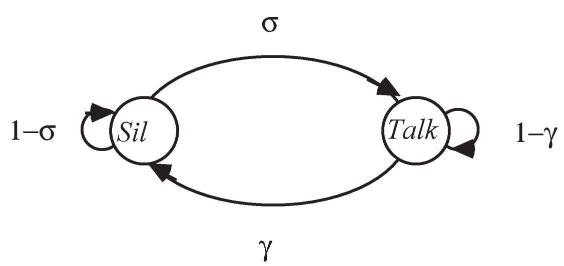

Figure 1. The voice model.

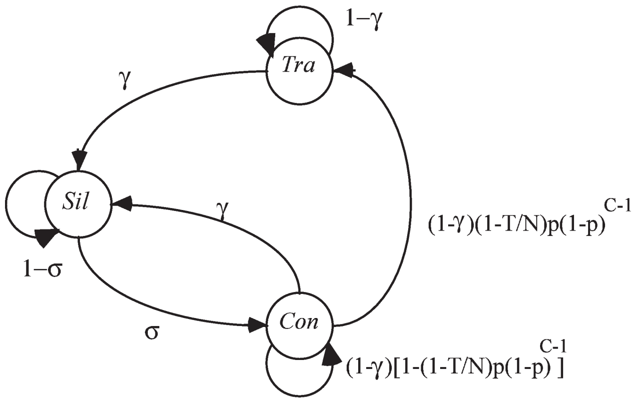

Figure 2. PRMA terminal state transition diagram.

constant and equal for all terminals. We model the system as discrete time Markov chain where, analogously to the voice model of section 3.1, we consider as unit of time the slot duration τ. In order to derive the system model, we first characterize and model the behavior of a single terminal.

As shown in figure 2, we assume that a terminal is always in one of the following states:

Sil the silent state,
Con the contending state,
Tra the transmission state.

A terminal is in state *Sil* during silent periods. When a talkspurt begins, the terminal leaves state *Sil* and enters state *Con*. The probability of this transition is σ in each time slot. In state *Con*, a terminal contends to obtain a reservation. If a talkspurt ends before a reservation has been obtained, the terminal leaves state *Con* and returns to state *Sil* without having transmitted any packet. The probability of this event is γ in each time slot.

To obtain a reservation and begin transmission, the following favorable conditions must be met: the talkspurt does not end during the current slot, the slot is not reserved, the terminal has permission to transmit and no collision occurs with packets of other contending terminals. Because these events are independent, we can write

$$\begin{aligned}&\Pr\{\text{transition from state } \mathit{Con} \text{ to state } \mathit{Tra}\}\\&\quad= \Pr\{\text{talkspurt does not end}\}\Pr\{\text{available time slot}\}\\&\qquad\times\Pr\{\text{permission to transmit}\}\Pr\{\text{no collisions}\}.\end{aligned}$$

From the voice model and PRMA system definition,

$$\begin{aligned}\Pr\{\text{talkspurt does not end}\} &= 1-\gamma,\\ \Pr\{\text{permission to transmit}\} &= p,\\ \Pr\{\text{no collisions}\} &= (1-p)^{C-1},\end{aligned}$$

where C denotes the current number of contending terminal.

For simplicity, we assume that the probability of an available time slot is given by the fraction of free time slots:

$$\Pr\{\text{available time slot}\} = \frac{N-T}{N} = 1-\frac{T}{N}, \qquad (2)$$

where T denote the current number of terminals in transmission.

Upon a successful transmission, a terminal transits from state *Con* to state *Tra*, where it stays as long as it has packets for transmission. After the terminal sends the last packet of the current talkspurt, it releases the reservation by leaving the slot empty and enters state *Sil*. Here, for simplicity of analysis, we assume that the probability of this transition is γ. This amounts to approximate the interval of time a terminal spends in transmission with a geometrically distributed random variable of mean $1/\gamma$, i.e., we assume it has the same distribution of a talkspurt duration. For the sake of continuity we postpone the motivation and discussion of this approximation later in this section.

The evolution of a single terminal among the three possible states is summarized in figure 2.

Given the state-dependent model of a single terminal, we can now derive the model for the PRMA system. We model the PRMA voice system as a discrete time Markov process

$$X = \{X_n = (S_n, C_n, T_n) \mid n \geqslant 0\}$$

with system state space $\mathbf{\Omega}$, and one-step transition probability matrix $\mathbf{P}$. S_n, C_n and T_n denote the number of terminals in state *Sil*, *Con* and *Tra* at time n, respectively (S_n is unnecessary because the three values always sum to M, but is retained for clarity). The state space $\mathbf{\Omega}$ is

$$\mathbf{\Omega} = \{(s,c,t) \mid s,c,t \geqslant 0,\ s \leqslant M,\ t \leqslant N,\ c = M-t-s\}.$$

The number of states is $(N+1)(M-N/2+1)$.

The entries of one-step transition probability matrix $\mathbf{P}$ are:

$$\Pr\{X_{n+1} = (s',c',t') \mid X_n = (s,c,t)\} = \sum_{\substack{s+i-j+k=s'\\ c+j-k-h=c'\\ t-i+h=t'}} \alpha_{ijkh},$$

where

$$\begin{aligned}\alpha_{ijkh} = {}&\Pr\{i \text{ transmitting terminals exit to silent state}\}\\&\times\Pr\{j \text{ silent terminals begin to contend}\}\\&\times\Pr\{k \text{ contending terminals return to silent state}\\&\qquad\text{and } h \text{ terminals get a reservation}\\&\qquad\text{and begin transmission}\}. \qquad (3)\end{aligned}$$

Based on the model of figure 2, we obtain the following expressions for the different terms in (3):

$$\Pr\{i \text{ transmitting terminals exit to silent state}\} = \binom{t}{i}\gamma^i(1-\gamma)^{t-i}, \qquad (4)$$

$$\Pr\{j \text{ silent terminals begin to contend}\} = \binom{s}{j}\sigma^j(1-\sigma)^{s-j}, \qquad (5)$$

$$\Pr\{k \text{ contending terminals return to silent state and } h \text{ terminals get a reservation and begin transmission}\}$$
$$= \binom{c}{k}\gamma^c(1-\gamma)^{c-k} \times \begin{cases} 1-\left(1-\frac{t}{N}\right)(c-k)p(1-p)^{c-k-1}, & h=0,\\ \left(1-\frac{t}{N}\right)(c-k)p(1-p)^{c-k-1}, & h=1. \end{cases} \quad (6)$$

The stationary probability distribution

$$\boldsymbol{\pi} = [\pi_{(s,c,t)}](s,c,t) \in \Omega,$$

can be computed from **P** by using standard techniques. Denote S, C and T the stationary number of terminals in each state. From the stationary distribution vector $\boldsymbol{\pi}$, we can compute the stationary distribution for the system variables S, C and T:

$$\begin{aligned} p_S(k) &= \Pr\{S=k\} = \sum_{(s,c,t)\in\Omega, s=k} \pi_{(s,c,t)},\\ p_C(k) &= \Pr\{C=k\} = \sum_{(s,c,t)\in\Omega, c=k} \pi_{(s,c,t)},\\ p_T(k) &= \Pr\{T=k\} = \sum_{(s,c,t)\in\Omega, t=k} \pi_{(s,c,t)},\\ &k = 0,\dots,M, \end{aligned} \quad (7)$$

as well as the expected values:

$$\begin{aligned} \mathrm{E}[S] &= \sum_{k=0}^{M} k p_S(k),\\ \mathrm{E}[C] &= \sum_{k=0}^{M} k p_C(k),\\ \mathrm{E}[T] &= \sum_{k=0}^{N} k p_T(k). \end{aligned} \quad (8)$$

From these results we can derive global system performance measures as the *system throughput*, *utilization* and *access delay*. We define the throughput as the average number of transmitted packets per frame and the system utilization as the fraction of slots per frame used to transmit packets. From the above analysis, it follows that the throughput is given by $\mathrm{E}[T]$ and the system utilization by $\mathrm{E}[T]/N$.

We define the access delay W as the expected value of the time t_c to obtain a reservation, that is equal to the average interval of time a terminal spends in contention. By Little's results,

$$W = \frac{\mathrm{E}[C]}{\mathrm{E}[S]\sigma} = \mathrm{E}[C]\frac{\gamma+\sigma}{M\gamma\sigma}, \quad (9)$$

where $\mathrm{E}[S]\sigma$ represent the steady state rate at which terminals enter (and exit) the contending state and $\mathrm{E}[C]$ the mean state population.

We end this section by discussing the choice of assuming the interval of time a terminal spends in transmission geometrically distributed with mean $1/\gamma$. Clearly, this assumption is an approximation of the real system behavior and amounts to ignore in the model that: (1) by system definition, a terminal stays in transmission for an interval of time that is a multiple of N slots; (2) since a terminal can store packets in the buffer during contention, the time it actually spends in transmission is function of the time previously spent in contention, and not independent, as we implicitly assume. Nevertheless, the above assumption can be justified with the following two observations:

1. Since the mean duration of a talkspurt is much longer than the frame duration (typically 1 s compared to few tens of milliseconds [3,4,9]), the exact distribution is well approximated by the adopted geometric distribution.
2. Because of the stringent delay requirement, the buffer is usually very small with respect to the average number of packets per talkspurt ($B = 1, 2$ in [3,4,6–9]). This suggests that the error introduced by neglecting the additional time required to transmit the buffered packets should be minimal. Indeed, our simulation study has revealed that under typical system parameters [9], for buffer sizes ranging from $B = 1$ up to $B = 10$ (corresponding to a maximum holding time ranging from 16 ms to 160 ms, i.e., up to $1/6$ of a mean talkspurt length) the system variables distributions are hardly affected by the value of the buffer size unless we consider highly congested systems. The insensitivity of the distributions in the above range allows us to ignore the effect of buffering in the PRMA system model.

4. Packet dropping analysis

Terminals drop voice packets delayed beyond the maximum holding time $D_{\max}$. Therefore, the performance of a PRMA voice system is characterized by its packet loss measures. In [4,9], the packet dropping probability, defined as the fraction of discarded packets, has been considered to characterize the packet loss performance of a PRMA system. However, voice quality is very sensitive to the loss of consecutive packets. For example, consider a packet loss probability of 1% for a typical voice application. Over, say, 100,000 voice packets, there is a considerable difference in the user's perception of quality of voice if one packet out of 100 is lost rather than 100 consecutive packets (same order of the average number of packets per talkspurt) out of 10,000. Hence, the knowledge of the distribution of the number of consecutive lost packets provides a better characterization of the quality of service provided by the PRMA system.

In this section we study packet dropping in the PRMA voice system. We evaluate the packet dropping probability as well as the distribution of lost packets within a talkspurt.

Observe that the latter is actually the distribution of the number of *consecutively* lost packets per talkspurt, since packets are dropped from the head of the talkspurt as soon as their delay exceeds $D_{\max}$.

Consider a reference terminal, say M. At the beginning of a talkspurt, the terminal generates the first packet and immediately enters the contending state. During contention, the terminal drops packets that have been held in the buffer for more than $D_{\max}$ slots. Moreover, the terminal drops all the packets still held in the buffer in the case the current talkspurt ends before a reservation has been obtained. During transmission, instead, since packets are generated and transmitted at the same rate, no packets will be discarded.

In order to calculate the number of lost packets by terminal M, during a contention period of length t_c, we need to distinguish between the case where the terminal eventually starts transmission and the case where it returns to the silent state before starting transmission. In the first case, no packet is lost if $t_c \leqslant D_{\max}$; for $t_c > D_{\max}$ one packet is lost plus one for each additional frame spent in contention. Denote n_l the number of lost packets. The following expression holds

$$n_l = \begin{cases} 0, & \text{if } 1 \leqslant t_c \leqslant D_{\max}, \\ k, & \text{if } D_{\max} + N(k-1) < t_c \leqslant D_{\max} + Nk, \\ & \quad k = 1, 2, \ldots. \end{cases} \tag{10}$$

In the second case, when a talkspurt ends before the terminal has obtained a reservation, all talkspurt packets are dropped and the whole message is lost. Since packets are generated at frame rate, the number of lost packets is

$$n_l = \left\lceil \frac{t_c}{N} \right\rceil. \tag{11}$$

To evaluate packet dropping performance measures, we need to model the behavior of terminal M during congestion and its interaction with the rest of the system. To this purpose we formulate the following model of a single terminal in contention.

Let terminal M be in the contending state and S'_n, C'_n and T'_n the number of the terminals $1, \ldots, M-1$, in each of the possible three states at time n. We model the single terminal in contention by a discrete time Markov chain

$$X' = \{ X'_n = (S'_n, C'_n, T'_n) \mid n \geqslant 0 \}$$

with state space $\boldsymbol{\Omega}'$, and one-step transition probability matrix $\mathbf{P}'$. The state space $\boldsymbol{\Omega}'$ is

$$\boldsymbol{\Omega}' = \{ (s', c', t') \mid s', c', t' \geqslant 0,\ s' \leqslant M-1,\ t' \leqslant N,\ c' = M - 1 - t' - s' \} \cup \{Sil\} \cup \{Tra\},$$

where $\{Sil\}$ and $\{Tra\}$ are absorbing states, and (s', c', t') are the possible configurations assumed by the other $M-1$ terminals during the contention period of the isolated terminal. The isolated terminal enters the contending state with the other $M-1$ terminals in one of the states (s', c', t'), according to an initial probability distribution; then, it keeps on contending with other terminals until it absorbs either in the *Sil* state (corresponding to a talkspurt termination before a reservation has been obtained) or in the *Tra* state (corresponding to the start of the transmission). The resulting number of states is $(N+1)(M - N/2) + 2$. The one step probability matrix is

$$\mathbf{P}' = \left(\begin{array}{c|cc} \widetilde{\mathbf{P}} & \mathbf{c}_{\mathrm{Sil}} & \mathbf{c}_{\mathrm{Tra}} \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right),$$

where $\widetilde{\mathbf{P}} = [\widetilde{P}_{(s'_1, c'_1, t'_1)(s'_2, c'_2, t'_2)}]$, (s'_1, c'_1, t'_1) and $(s'_2, c'_2, t'_1) \in \boldsymbol{\Omega}' - \{Sil, Tra\}$, is a substochastic matrix, describing the transition probabilities among the transient states, and $\mathbf{c}_{\mathrm{Sil}} = [c_{(s',c',t')\mathrm{Sil}}]$ and $\mathbf{c}_{\mathrm{Tra}} = [c_{(s',c',t')\mathrm{Tra}}]$, $(s', c', t') \in \boldsymbol{\Omega}'$, are column vectors, describing the one-step absorbing probabilities to state *Sil* and *Tra*, respectively. The entries for the matrix $\widetilde{\mathbf{P}}$ can be easily derived as made in section 3 for the overall PRMA model, by considering that $\widetilde{\mathbf{P}}$ describes the state transitions of a PRMA system with $M-1$ terminals plus one always in contention. The entries for $\mathbf{c}_{\mathrm{Sil}}$ and $\mathbf{c}_{\mathrm{Tra}}$ can be calculated from the probabilities of the events that terminate a contention period, as explained above.

It is easy to realize that the time t_c spent in contention is equal to the time up to absorption for the process X'. Hence, we can derive the expression for the distribution of the number of lost packets from the distribution of the time up to absorption of the process X':

$$\begin{aligned} \Pr\{n_l \leqslant k\} &= \Pr\{n_l \leqslant k,\ \text{reservation obtained}\} \\ &\quad + \Pr\{n_l \leqslant k,\ \text{no reservation obtained}\} \\ &= \Pr\{X'_{D_{\max}+Nk} = Tra\} \\ &\quad + \Pr\{X'_{Nk} = Sil\}. \end{aligned} \tag{12}$$

Conditioning on the initial state, we get the following expression:

$$\begin{aligned} \Pr\{n_l \leqslant k\} = \sum_{(s',c',t') \in \boldsymbol{\Omega}'} & \pi^{(0)}_{(s',c',t')} \\ & \times \Pr\{X'_{D_{\max}+Nk} = Tra \mid X'_0 = (s', c', t')\} \\ & + \Pr\{X'_{Nk} = Sil \mid X'_0 = (s', c', t')\}. \end{aligned} \tag{13}$$

The packet dropping probability, P_{drop} is defined as the fraction of dropped packets [9]. From the voice model, the mean number of packets in a talkspurt is $1/(1-(1-\gamma)^N) \approx 1/(\gamma N)$; hence, the expression for the packet dropping probability is

$$P_{\mathrm{drop}} = \mathrm{E}[n_l]\big(1 - (1-\gamma)^N\big) \approx \mathrm{E}[n_l]\gamma N, \tag{14}$$

where $E[n_l]$ can be simply derived from the distribution (13).

In order to compute the distribution of the number of lost packets given by (13), we have to compute both the conditional probabilities $\Pr\{X'_n \mid X'_0\}$ for $n = D_{\max} + Nk$ and $n = Nk$, and the initial probability vector $\boldsymbol{\pi}^{(0)} = [\pi^{(0)}_{(s',c',t')}]$, $(s', c', t') \in \boldsymbol{\Omega}'$, that gives the probability of entering the contending state with the other terminals in state (s', c', t'). The conditional probabilities $\Pr\{X'_n \mid X'_0\}$

can be obtained from the entries of the n-step transition probabilities matrix $\mathbf{P}'^{n}$. The initial probability vector $\boldsymbol{\pi}^{(0)}$ can be obtained as follows. Let us observe that, because a single terminal (figure 2) enters or leaves the *Sil* state independently of the state of the rest of the system, it actually gets a random look to the state of the PRMA system with itself removed, i.e., with one less terminal. Hence, we have $\pi^{(0)}_{(s',c',t')} = \pi_{(s',c',t')M-1}$, $(s',c',t') \in \Omega'$, where the subscript $M-1$ denotes the steady state probability of the overall PRMA system with $(M-1)$ terminals. Thus the initial probability vector for the model of the isolated contending terminal can be obtained from a PRMA system with one less terminal by solving the model of section 3.

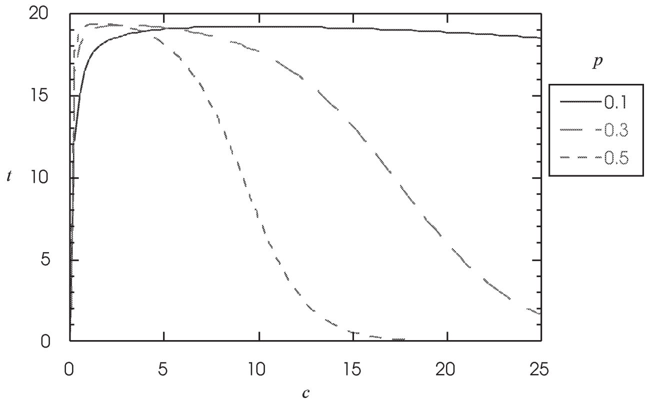

Figure 3. Equilibrium contours.

5. Stability analysis

PRMA system has been shown to exhibit instability behavior, a common phenomenon among all multiple access contention protocols as the offered load increases [5,9–11]. In this section, we study the stability of the PRMA system using equilibrium point analysis (EPA)[11,12]. Our approach differs from the analogous study in [9] in that we extend to the PRMA protocol the approach for the stability analysis of the ALOHA protocol [5]. Specifically, we extend the notion of channel *load line* given in [5] by introducing the *PRMA load line* concept; this allows us to obtain a simple graphical characterization of the PRMA system stability.

For a Markovian model, an equilibrium point is defined as the values of the state variables for which the expected change in each state variable is zero. When the system is at an equilibrium point the expected rate at which the system leaves the state is equal to the expected rate at which the system enters the state. A system is said to be stable if it has only one equilibrium point. In a stable system, the system variables will fluctuate around the equilibrium point values that represent the "operating point" of the system. In this case, as several studies indicate [5,9–11], the equilibrium values of the state variables are expected to closely approximate their mean values. Conversely, a system is said to be unstable if it has multiple equilibrium points. In an unstable system, the system variables will oscillate between the multiple equilibrium points values [5,9,10]. In this case, even if the equilibrium values cannot be used for an evaluation of the system performance, they provide an indication of system dynamics.

For a PRMA system, let s, c and t be the equilibrium number of terminals in silence, contention and transmission state, respectively. At equilibrium, the rate at which the system state changes is zero. This corresponds to consider equal the inflow and the outflow from each state [11].

From the model of figure 2, we derive the following equation for the equilibrium at state *Tra*:

$$(1-\gamma)\left(1-\frac{t}{N}\right)cpu(c) = t\gamma, \tag{15}$$

where

$$u(c) = \begin{cases} 1, & c < 1, \\ (1-p)^{c-1}, & c \geqslant 1. \end{cases}$$

We can rewrite (15) to obtain an explicit expression of t as function of c:

$$t = \frac{N(1-\gamma)cpu(c)}{N\gamma + cpu(c)(1-\gamma)}. \tag{16}$$

Equation (16) defines an *equilibrium contour* in the (c,t)-plane, that represents the locus of points where the rate at which contending terminals begin transmission (left hand side of (15)) equals the rate at which transmitting terminals end transmission (right hand side of (15)). In figure 3 we show some of these contours in the (c,t) plane for a particular PRMA system ($N = 20$, $\gamma = 0.0008$, $\sigma = 0.0006$ [9]) for different permission probability values.

Similarly, from the model of figure 2 we obtain the following equation for the equilibrium at state *Sil*:

$$\sigma s = \gamma(M - s), \tag{17}$$

where we use the fact $s+c+t = M$. From (17) we obtain the equilibrium number of silent terminals:

$$s = M\frac{\gamma}{\gamma+\sigma}. \tag{18}$$

Substituting $s = M - c - t$ in (18), we obtain an additional equation relating c and t,

$$t = M\frac{\sigma}{\gamma+\sigma} - c \tag{19}$$

that we call the *PRMA load line*. This intercepts both axis at the point $M(\sigma/(\gamma+\sigma))$, and represents the locus of (c,t)-points for which the number of silent terminals is in equilibrium.

The intersections of the PRMA load line with the equilibrium contour represent the system equilibrium points. A PRMA system is said to be *stable* if it has a single equilibrium point, i.e., if the PRMA load line intersects (non-tangentially) the equilibrium contour in exactly one place. Otherwise the PRMA system is said to be *unstable*.

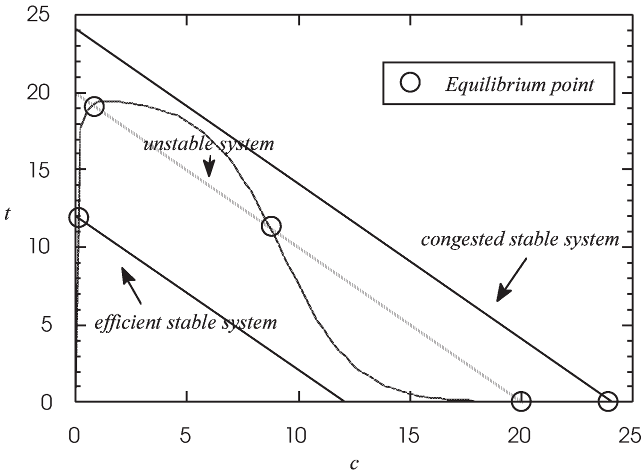

Figure 4. Stability of PRMA system.

According to the relative position of the load line with respect to the contour plot, we can distinguish three different cases as illustrated in figure 4. Here we fix the equilibrium contour and consider three different load lines obtained, from left to right, by increasing the number of terminals (see (19)). Because of the single intersection with the equilibrium contour, the two solid load lines represent stable PRMA systems: the first one corresponds to an efficient PRMA system with a high equilibrium number of transmitting terminals; the second one represents a highly congested system, with a very small equilibrium number of transmitting terminals. Finally, consider the dotted load line. This intersects the contour plot in three points and, according to the above definition, corresponds to an unstable system. In this case, the system oscillates between its two l*ocally stable equilibrium points* [5], corresponding here to the equilibrium points with the smallest and largest value of c. This indicates that the system periodically experiences periods of very poor performance.

6. Numerical examples

In this section, we present numerical examples to illustrate the results of the previous sections. We study the performance and the stability of a PRMA system and their dependence on system parameters. Our focus is on evaluating the packet dropping performance measures and analyze their impact on the quality of service provided by a PRMA voice system. The accuracy of our analytical results is verified by comparison with simulation. The system parameters we use in our examples (corresponding to the values used in [9]) are summarized below, together with the corresponding values of the model parameters:

Frame duration	16 ms,	
Number of slots per frame N	20,	
Maximum holding time	32 ms	($D_{\max} = 40$ slots, $B = 2$),
Mean talkspurt duration t_1	1 s	($\gamma = 0.0008$),
Mean silence duration t_2	1.35 s	($\sigma = 0.0006$).

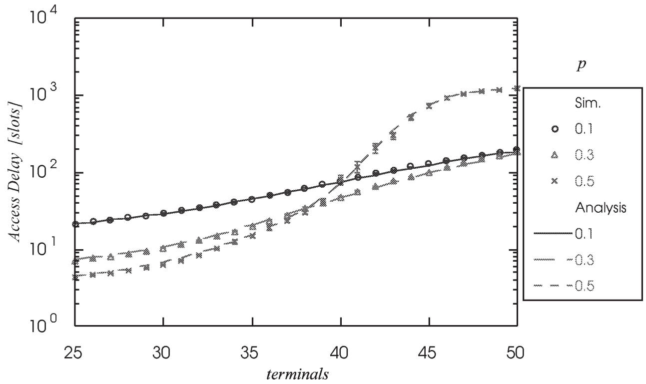

Figure 5. Access delay.

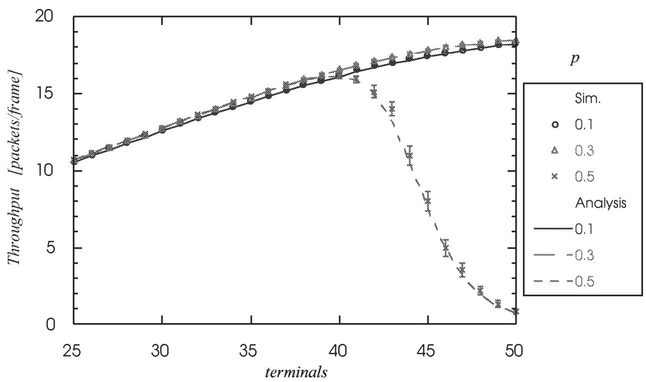

Figure 6. Throughput.

In figures 5 and 6, we plot the access delay and system throughput, respectively, as function of the number of terminals for different values of the permission probability p. Both analytical and simulation results are shown. The simulation results have been obtained by running each simulation for a duration of one million frames. In all the figures, we show the 95% confidence interval. Observe that, although we introduced approximations in our model, as we have discussed in section 3, the analytical results are very accurate over the entire considered range.

The access delay increases with the number of terminals. In the considered range, increasing the permission probability from $p = 0.1$ to $p = 0.3$ allows terminals to contend more frequently, thus decreasing the time spent in contention, and allowing higher throughput. For example, with $M = 25$, the access delay for $p = 0.1$ is three times larger (21 slots compared to only 7 slots for $p = 0.3$). By further increasing the permission probability to 0.5, system performance can be further improved if the number of terminals is small. On the other hand, if the number of terminals exceeds a given threshold, the high permission probability leads to excessive collisions and congestion, and performance dramatically deteriorates. Observe that for $p = 0.5$, as M exceeds 35, the access delay rapidly increases and the throughput quickly drops to zero.

The beginning of the rapid performance degradation is associated with the onset of system instability and can be

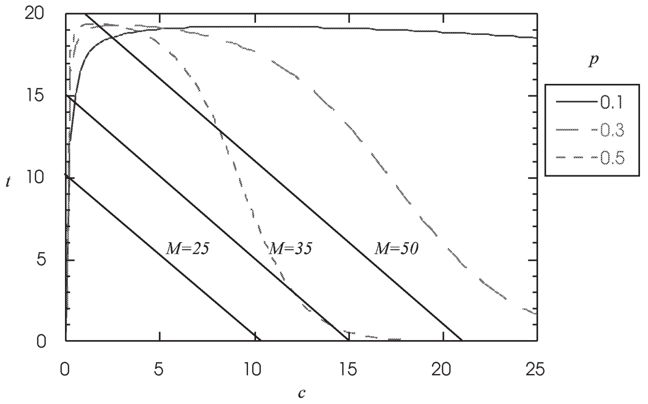

Figure 7. PRMA system stability analysis.

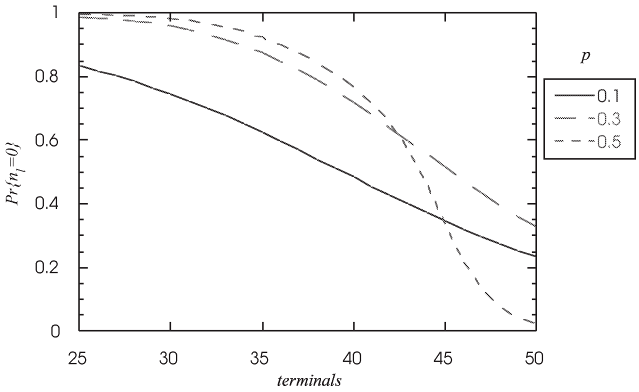

Figure 9. Probability that a talkspurt suffers no loss.

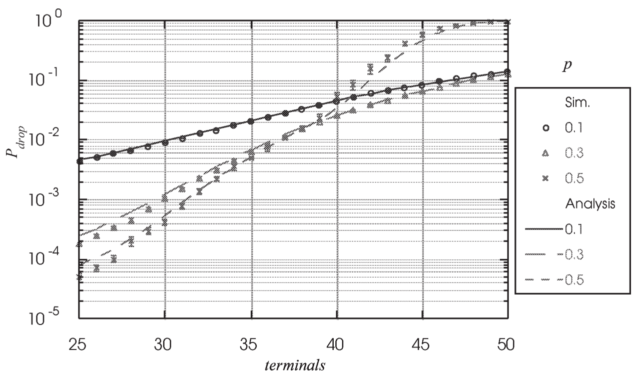

Figure 8. Packet dropping probability.

studied by means of stability analysis. To this purpose, we plot in figure 7 the system equilibrium contours for different values of the permission probability *p* and the load lines corresponding to $M = 25$, 35 and 50. The system is stable when the number of terminals is small under all the considered values of p. As the number of terminals increases, the system becomes unstable for $p = 0.5$ and $M = 35$. Indeed, the load line corresponding to $M = 35$ intercepts the equilibrium contour corresponding to $p = 0.5$ in three points. One of these corresponds to a stable, highly congested equilibrium point. As simulation results have shown, the system oscillates between its two stable equilibrium points, indicating that the system experiences periodically poor performance. By further increasing the number of terminals, the fraction of time spent in the congested state increases, and the system performance rapidly deteriorates.

Next, we analyze packet dropping performance measures, and first consider the packet dropping probability. In figure 8, we plot the packet dropping probability as function of the number of terminals for different values of the permission probability p. As shown in section 4, the packet dropping performance is closely related to the length of the contention period: the longer the waiting time to obtain a reservation, the higher the suffered packet loss. For light loads, we can observe that the packet dropping probability decreases as terminals are allowed to contend more frequently. This mechanism becomes ineffective for high loads, because high permission probabilities lead to excessive collisions and increasing delay to obtain a reservation, that results into a high fraction of dropped packets.

The packet dropping probability gives the fraction of packets that are lost over an infinite horizon, but provides no information about how losses occur within talkspurts. Knowledge of the latter is important because it better describes the voice quality as perceived by a user. To this end, we now turn our attention to distribution analysis.

As a first characterization of the loss performance within a talkspurt, we consider the probability that a talkspurt suffers no loss, plotted in figure 9. As expected, this probability decreases as the traffic increases and exhibits the same qualitative behavior of the other performance measures as p varies. Comparing figures 8 and 9, observe that in correspondence of small values of P_{drop}, loss is localized in a small fraction of the talkspurts, whereas the large majority does not suffer any loss and quality of service degradation. For example, for $p = 0.1$ the percentage of talkspurts that suffer no loss reaches 84%, and it can be as high as 98.5% and 99.5% for $p = 0.3$ and $p = 0.5$, respectively. This indicates, however, that within talkspurts that do suffer loss, the performance can be definitely worse than the one predicted by the value of P_{drop}. To illustrate this point, we fix the number of terminals to $M = 36$, and study the packet loss distribution for $p = 0.3$ and $p = 0.5$. We choose the value $M = 36$ because, assuming that 1% of dropped packets is acceptable [4,6,9], it represents for both the above values of the permission probability the “capacity” of the PRMA system, defined as the maximum number of terminals that can be efficiently supported, while keeping the packet dropping probability below the given loss constraint.

In figure 10, we plot the complementary distribution function $\Pr\{n_l > n\}$ of the number of lost packets in a talkspurt for $p = 0.3$ and $p = 0.5$. For these two values of p, the corresponding packet dropping probability is 0.0094 and 0.0077, respectively. Hence, the use of an average measure such as P_{drop} suggests that the better configuration is obviously represented by the higher value of the permission probability that ensures 19% less of dropped packets. With the help of figure 10, we now study in more details

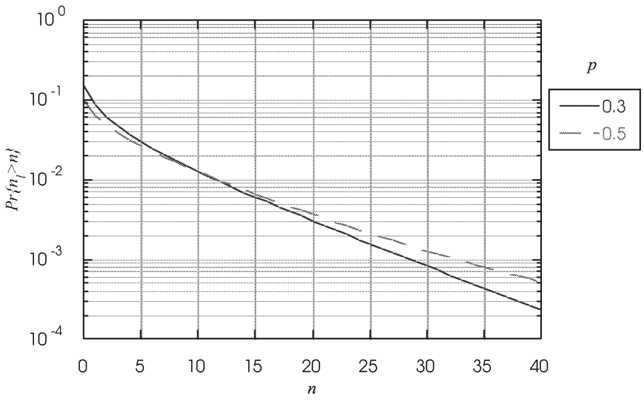

Figure 10. Packet dropping distribution ($M = 36$).

how losses occur. First, observe that the percentage of talkspurts that suffer no loss is reasonably high in both cases: 85% for $p = 0.3$ and 90% for $p = 0.5$. This means that all the losses occur in the remaining 15% and 10% of the talkspurts, respectively. Therefore, in a talkspurt that suffers loss, the fraction of dropped packets is much higher (approximately 6 times ($p = 0.3$) and 10 times ($p = 0.5$) larger than P_{drop}). This suggests also that the probability of losing several consecutive packets can be high, despite the small value of P_{drop}. Observe, indeed, that the probability of dropping more than 10 consecutive packets, (corresponding to 160 ms of voice, $1/6$ of the average duration of a talkspurt) is in both cases equal to 1.26%, i.e., the probability that a talkspurt suffers a (relative) long clipping is higher than the packet dropping probability itself. Furthermore, if we condition this probability on the event that the talkspurt drops packets, i.e., if we consider $\Pr\{n_l > 10 \mid n_l > 0\}$, then the two probabilities become as high as 8.36% ($p = 0.3$) and 12.66% ($p = 0.5$). This clearly shows that the performance degradation within a talkspurt can be much higher than the one predicted by average measures.

To further illustrate the importance of distribution analysis, we now compare the two distribution curves in figure 10. Observe that the tail of the distribution for $p = 0.3$ has a faster decay rate. This indicates that for $p = 0.3$, packet dropping mostly occurs as loss of just few packets, while it is very unlikely that several consecutive packets are lost. Compared to the case for $p = 0.3$, for $p = 0.5$ the probability of losing just few packets is smaller, while it is larger the one of losing several consecutive packets. Therefore, if we assume that perceptible voice degradation is only caused by the loss of many consecutive packets, the above analysis indicates that the smaller value of the permission probability yields better performance despite the higher fraction of lost packets. This is no surprise since we already know that for $p = 0.5$ the system exhibits unstable behavior with periods of high congestion. Obviously, such kind of finer analysis is impossible by using average values only.

7. Conclusions

In this paper, we have presented a thorough analysis of the PRMA voice protocol. We have developed a Markov model of the PRMA voice system, the size of which allows direct evaluation of the system performance. The system stability has also been characterized. To analyze the packet dropping process we have studied the dynamics of a single terminal. We have evaluated packet loss performance measures as the packet dropping probability and the distribution of the number of dropped packets within a talkspurt. In our numerical study, we have shown that the latter provides a better characterization of the quality of service as perceived by a user. In particular, the results on the distribution of the number of dropped packets in a talkspurt have revealed that losses are highly correlated. Indeed, within a talkspurt, several consecutive packets can be lost with severe performance degradation, even when the packet dropping probability is kept below small values (e.g., 1%).

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