Source Time Scale and Optimal Buffer/Bandwidth Trade-off for Regulated Traffic in an ATM Node*

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Abstract

In this paper, we study the problem of resource allocation and control for an ATM node with regulated traffic. Both guaranteed lossless service and statistical service with small loss probability are considered. We investigate the relationship between source characteristics and the buffer/bandwidth trade-off under both services.

Our contributions are the following. For guaranteed lossless service, we find that the optimal resource allocation scheme suggests a time scale separation of sources sharing an ATM node with finite bandwidth and buffer space, with the optimal buffer/bandwidth trade-off is determined by the sources' time scale. For statistical service with a small loss probability, we present a new approach for estimating the loss probability in a shared buffer multiplexor with the so called "extremal" on-off, periodic sources. Under this approach, the optimal resource allocation for statistical service is achieved by maximizing both the benefits of buffering sharing and bandwidth sharing. The optimal buffer/bandwidth trade-off is again determined by time scale separation.

1 Introduction

Resource allocation is an extremely challenging and important problem in the design and control of high-speed networks such as ATM networks. The problem is particularly complicated by the need to support *Quality-of-Service* (QoS) guarantees for a variety of applications with very diverse traffic characteristics.

In [5], a new approach to resource allocation in an ATM node with fixed bandwidth and a finite, shared buffer is presented. In this approach, an ATM node is modeled by a shared buffer multiplexor with the so-called "extremal" on-off, periodic arrival processes, which account for the worst-case stochastic behavior (as proved for the bufferless multiplexor in [8, 14]). The ingenuity of the approach is the reduction of the two-resource (*i.e.*, buffer and bandwidth) allocation problem to a single-resource allocation problem, *i.e.*, the known problem of estimating loss probability of a bufferless multiplexor. This reduction is made possible by introducing the concept of a virtual buffer/trunk system and establishing the "exchangeability" of buffer and bandwidth. Based on the analytical results, a qualitative theory is then described which provides many insights in call admission control.

Motivated and inspired by the work in [5], we study source characteristics and their impact on buffer/bandwidth trade-off in the design and control of an ATM node. The starting point of our approach is the examination of optimal resource allocation schemes for guaranteed lossless service. We find that under such service, the optimal resource allocation scheme for an ATM node where each virtual circuit has its own allocated bandwidth and buffer space with no resource sharing (referred to as a lossless segregated system) is no different from that for an ATM node with all virtual circuits sharing the resources (*i.e.*, a lossless multiplexing system). Hence, in this case, there are no benefits in resource sharing, and the two systems are effectively equivalent. We also find that the optimal resource allocation scheme suggests an interesting separation of time scales among sources sharing an ATM node with finite bandwidth and buffer space, with the optimal buffer/bandwidth trade-off being determined by this time scale separation. Sources are classified as having either "fast" or "slow" time scales, reflecting the efficacy of either buffer sharing or bandwidth sharing among the sources.

For statistical service where a small loss probability is allowed, we derive a new approach to estimate the loss probability using our results for the optimal buffer/bandwidth trade-off obtained for lossless service. By giving a new interpretation to the virtual trunk/buffer systems introduced in [5], we are able to transform the two-resource allocation problem into two independent single-resource allocation problems. The best buffer/bandwidth separation is explored by optimizing resource allocation along the optimal buffer/bandwidth trade-off curve. Through numerical examples, we demonstrate that source time scales also have a major impact on the optimal resource allocation under statistical service, and the optimal buffer/bandwidth trade-off is again reflected by the source time scale separation.

Our work differs from [5] in several aspects. First, our perspectives on resource allocation and control problems are somewhat different. The authors in [5] are primarily interested in call admission control. This is reflected in their fixing the system bandwidth C and buffer space B. Resource allocation to each source is independent of the sources' characteristics. In our approach, we fix one resource (bandwidth C) and find the optimal allocation of the other resource (buffer space B). Furthermore, resource

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allocation is made according to the time scale separation of the system and the source's own time scale. Due to this difference in perspectives, we are able to study the role of source time scale and investigate optimal buffer/bandwidth trade-off for both lossless service and statistical service with a given loss probability. We are also able to explore the maximal benefits of both buffer sharing and bandwidth sharing. This is important, as the efficacy of buffer sharing and that of bandwidth sharing for sources with different time scales are quite different. Numerical examples indicate that our approach provides a better estimate of the system loss probability than [5].

The remainder of this paper is organized as follows. We start with the optimal resource allocation problem for guaranteed lossless service in Section 2.In Section 3, we study the optimal resource allocation problem for statistical service with small loss probability. In Section 4, numerical examples are presented to illustrate the relationship between source time scale and buffer/bandwidth trade-off. The paper is concluded in Section 5.

2 Guaranteed Lossless Service

The starting point of our study is the analysis of the optimal resource allocation scheme for guaranteed lossless service. Consider an ATM node with a total amount of bandwidth C and buffer space B. Suppose there are N virtual circuits sharing the node. Each virtual circuit is associated with a traffic source that is leaky bucket regulated [12]. We consider the following two scenarios. In the first scenario, each virtual circuit is allocated a fixed portion of the total bandwidth and buffer space with no resource sharing among the virtual circuits. We call this system *lossless segregated system* . In the second scenario, the resources are shared among the virtual circuits. We call such a system *lossless multiplexing system* . We are interested in optimal resource allocation schemes that, for given bandwidth C, minimize the buffer requirement while ensuring that no virtual circuits ever incur losses in the above scenarios. We show that for guaranteed lossless service, the optimal resource allocation schemes for both systems are the same. Hence in terms of resource requirements, the lossless multiplexing system is effectively equivalent to the lossless segregated system. First, we describe the regulated traffic sources.

A leaky bucket regulator is characterized by three parameters: the token rate ρ, the token bucket size σ and the peak rate P, where $P \geq \rho$. Let $A[\tau, \tau + t]$ denote the amount of traffic passing through the regulator in the time interval $[\tau, \tau + t]$. Then

$$A[\tau,\tau+t] \leq \mathcal{E}(t) := \min\{Pt, \sigma+\rho t\},\ \tau \geq 0,\ t \geq 0 \quad (1)$$

where $\mathcal{E}(t)$ is called the minimum envelope process for the regulated source [3]. It bounds the amount of traffic departing from the regulator during any time interval of length t.

Let T_{on} denote the maximum length of a peak rate burst, *i.e.*, $T_{on} = \frac{\sigma}{P-\rho}$. A traffic source which generates traffic at peak rate P for T_{on} time and switches to rate ρ for the rest of the time has a regulated traffic such that $A[0, t] = \mathcal{E}(t)$. We call such a traffic source *greedy*.

For the purpose of exposition, we assume that the N traffic sources are classified into J classes according to their regulator characterization, where all regulated sources in class j have the same leaky bucket parameters (ρ_j, σ_j, P_j), $1 \leq j \leq J$. There are K_j Class j sources, and $\sum_{j=1}^{J} K_j = N$. We assume that all classes have different peak rate burst length T_{on}. Without loss of generality, let $T_{on_1} > T_{on_2} > \cdots > T_{on_J}$. In order to have a stable system, we require that $\sum_{j=1}^{J} K_j\rho_j \leq C$. Also in order to avoid triviality, we assume that $C \leq \sum_{j=1}^{J} K_j P_j$.

2.1 Lossless Segregated System

We first consider the optimal resource allocation problem for the lossless segregated system. We fix the total bandwidth C for the system, and consider allocation schemes that minimize the total buffer space required to ensure that no virtual circuits incur any losses.

For $j = 1, \ldots J$, suppose each source in class j is allocated bandwidth c_j and buffer space b_j. Let $C_j = K_j c_j$ denote the total amount of bandwidth allocated to class j sources. The stability condition requires that $c_j \geq \rho_j$ for each j. In order to ensure that no losses occur for any virtual circuit, the amount of buffer space b_j allocated to a class j source is determined by the maximum queue length for each segregated virtual circuit j, *i.e.*,

$$b_j \geq \sup_{t\geq 0}\{\mathcal{E}_j[t] - c_j t\} = \sigma_j - T_{on_j}(c_j - \rho_j). \quad (2)$$

The overall buffer space required to ensure that no virtual circuits encounter losses is thus $B_{seg} = \sum_{j=1}^{J} K_j b_j$. This determines the buffer requirement under the segregated allocation scheme.

Given the linearity of (2), the optimal buffer allocation problem can be formulated as the following Linear Programming (LP) problem:

Problem Minimize
$B_{seg} = \sum_{j=1}^{J} K_j\sigma_j - \sum_{j=1}^{J} T_{on_j}(C_j - K_j\rho_j)$
subject to:
$\sum_{j=1}^{J} C_j = \sum_{j=1}^{J} K_j c_j \leq C,$
$\rho_j \leq c_j \leq P_j, \quad j = 1, \ldots, J.$

It is clear that the objective function decreases whenever bandwidth is taken from classes with smaller T_{on} and is allocated to classes with larger T_{on}. As a consequence, the optimal allocation scheme consists of allocating peak rate to as many classes with large T_{on} as possible without violating the constraint $\sum_{j=1}^{J} C_j \leq C$, while allocating only average rate to classes with small T_{on}. Formally, let k be the smallest index such that

$$\sum_{j=1}^{k-1} K_j P_j + \sum_{j=k}^{J} K_j\rho_j \leq C < \sum_{j=1}^{k} k_j P_j + \sum_{j=k+1}^{J} K_j\rho_j. \quad (3)$$

Then the optimal resource allocation scheme that results in the minimum buffer requirement for the given bandwidth C is as follows:

$$c_j = \begin{cases} P_j, & j = 1, \ldots, k-1, \\ \frac{C - \sum_{l\neq k} K_l c_l}{K_j}, & j = k, \\ \rho_j, & j = k+1, \ldots, J \end{cases} \quad (4)$$

and

$$b_j = \begin{cases} 0, & j = 1, \ldots, k-1, \\ \sigma_j - T_{on_j}(c_j - \rho_j), & j = k, \\ \sigma_j, & j = k+1, \ldots, J. \end{cases} \quad (5)$$

Note that $\sum_{j=1}^{J} C_j = \sum_{j=1}^{J} K_j c_j = C$, and the buffer requirement B_{seg} has the following closed-form expression in terms of the regulated source parameters

$$B_{seg} = \sum_{j=k+1}^{J} K_j \sigma_j + [K_k \sigma_k - K_k T_{on_k}(c_k - \rho_k)]. \quad (6)$$

2.2 Lossless Multiplexing System

We now consider the optimal resource allocation problem for the lossless multiplexing system. Again we fix the total bandwidth of the system which is shared by all virtual circuits, and determine the minimal buffer space required to guarantee no losses. This shared buffer multiplexing system is equivalent to a shared single queue serviced by a server of capacity C. Given the regulated traffic sources defined earlier, the maximum queue length of the system is given by the following expression

$$Q_{max} \leq \sup_{t \geq 0} \{ \sum_{j=1}^{J} K_j \mathcal{E}_j(t) - Ct \} \quad (7)$$

where the equality is attained when all sources are greedy and start at the same time. Hence in order to ensure that no losses occur in the system, a minimum buffer size $B_{mux} = Q_{max}$ is required.

We proceed to derive a closed-form expression for B_{mux} in terms of the parameters of the regulated sources. By substituting the expression for $\mathcal{E}_j(t)$ in (7), we readily obtain that the supremum is attained at the point $t_{max} = T_{on_k}$, where k is exactly as defined in (3). Hence, T_{on_k} has the following physical meaning: T_{on_k} is the time the system reaches its maximum queue length when all sources are greedy.

From (7), we derive that the minimum buffer requirement is

$$B_{mux} = \sum_{j=k+1}^{J} K_j \sigma_j + [K_k \sigma_k - T_{on_k}(C - \sum_{j=1}^{k-1} K_j P_j - \sum_{j=k}^{J} K_j \rho_j)]$$

Comparing this with (6), we observe that $B_{seg} = B_{mux}$. Hence the minimum buffer requirement for the lossless multiplexing system is exactly the same as for the lossless segregated system. Thus we can define $B_{min} = B_{seg} = B_{mux}$ as the buffer requirement for lossless service.

From (3), we observe that if, for $1 \leq j \leq J$, we define c_j and b_j as in (4) and (5), then $\sum_{j=1}^{J} K_j c_j = C$ and $\sum_{j=1}^{J} K_j b_j = B_{min}$. This observation provides the following interesting interpretation as to how the system resources are optimally shared among the regulated sources in the lossless multiplexing system. Specifically, when resources are optimally allocated in the lossless multiplexing system, the virtual circuits behave *as if each of them was allocated fixed bandwidth c_j and fixed buffer space b_j*, just as in the lossless segregated system. Hence the lossless multiplexing system can be effectively treated as if it were the lossless segregated system. This observation provides a motivation for the approach we take in Section 3 for studying resource allocation schemes under statistical multiplexing with small loss probability.

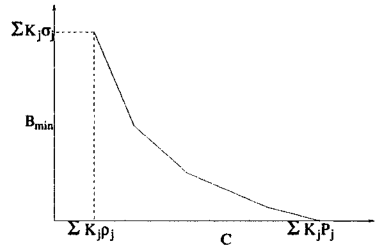

Figure 1: Optimal buffer/bandwidth trade-off curve.

2.3 Source Time Scale and Optimal Buffer/ Bandwidth Trade-off Curve

This far we have studied the resource allocation problem by fixing the bandwidth C. Now we consider the buffer/bandwidth trade-off for lossless service with a given set of regulated sources.

For any bandwidth C such that $\sum_{j=1}^{J} K_j \rho_j \leq C \leq \sum_{j=1}^{J} K_j P_j$, the index k defined in (3) is determined solely by the regulated source parameters and plays a key role in determining the minimal buffer requirement B_{min} (see (6)). Hence in order to study the buffer/bandwidth tradeoff, it suffices to study k as a function of C. From (3), we see that k is non-decreasing in C: $k = 1$ when $C = \sum_{j=1}^{J} K_j \rho_j$, and $k = J$ when $C = \sum_{j=1}^{J} K_j P_j$. As a consequence, from (4) and (5), we have that the allocated bandwidth c_j to class j sources is a non-decreasing function of C, whereas the buffer space b_j allocated to these sources is a non-increasing function of C. Therefore, the buffer requirement B_{min} is a decreasing function of C. Moreover, from (4) and (5), we have $\frac{dB_{min}}{dC} = -T_{on_k}$. Thus, the buffer requirement B_{min} is a piece-wise linear, decreasing convex function of C(see Figure 1). We call the curve (C, B_{min}) in Figure 1 the buffer/bandwidth trade-off curve for lossless service.

The optimal resource allocation scheme also suggests a taxonomy of the regulated sources according to their maximum peak rate burst length T_{on_j}, which we shall also refer to as the time scale of the regulated sources. Subsequently, we call the index k the source *time scale index* with respect to (C, B_{min}). Sources in class j are said to have either have "fast" time scale or "slow" time scale with respect to (C, B_{min}) according to whether $T_{on_j} < T_{on_k}$ or $T_{on_j} > T_{on_k}$. Under the optimal resource allocation scheme, we see that the most efficient way to accommodate "fast" time scale sources under lossless service is to allocate minimum amount of bandwidth (equal to their mean rates) and maximum buffer space (equal to their token bucket sizes), while the most efficient way to accommodate "slow" time scale sources is to allocate maximum bandwidth (equal to their peak rate) and zero buffer space.

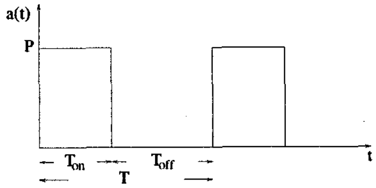

Figure 2: Extremal on-off, periodic curve.

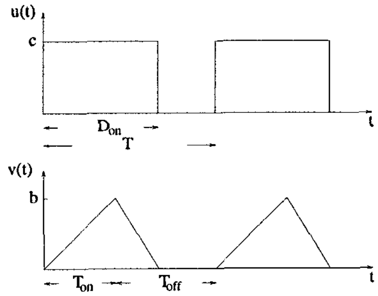

Figure 3: Buffer/bandwidth separation.

3 Statistical Service with Small Loss Probabilities

In the preceding section we established that under guaranteed lossless service, the optimal resource allocation is the same, regardless of whether resources are shared among sources. In this section, we study the benefits of resource sharing under statistical service where a small probability of loss, say, $10^{-9} \sim 10^{-7}$, is allowed, and investigate the buffer/bandwidth trade-off under such statistical service.

Once again, consider an ATM node with N virtual circuits. Each virtual circuits is associated with a leaky bucket regulated traffic source. Suppose the node has a total amount of bandwidth C and buffer space B, shared by all the sources. We assume that the system resources are not sufficient to provide guaranteed lossless service. In other words, the lossless service buffer requirement B_{min} for the given value of bandwidth C exceeds B. This implies that (C, B) lies below the buffer/bandwidth trade-off curve in Figure 1. In this section, we explore the possibility of exploiting statistical multiplexing gains by considering statistical service with small loss probabilities.

Statistical multiplexing gains can be extracted by exploiting the bursty nature and statistical independence of traffic sources [5]. To exploit the bursty nature of traffic sources regulated by leaky buckets with parameters (σ, ρ, P), we follow [5] and assume that the regulated sources after passing through leaky bucket regulators are extremal on-off, periodic processes (see Figure 2), where, when a source is active, it generates data at the peak rate P until the depletion of its token bucket; it then stays inactive until the token bucket is completely filled again. Use of such processes are justified to a large extent by the work of [4, 8, 13, 14]. In particular, such processes account for the *worst-case* statistical behavior in a bufferless multiplexor in the sense that they maximize the average loss rate [14] and the loss probability estimated by the Chernoff bound [8, 14].

Let $a(t)$ denote an extremal on-off, periodic departure process from a leaky bucket regulator with parameters (ρ, σ, P) as shown in Figure 2. Then the lengths of the on and off periods are $T_{on} = \frac{\sigma}{P-\rho}$ and $T_{off} = \frac{\sigma}{\rho}$. The source period is $T = T_{on} + T_{off} = P/\rho T_{on}$. Let S denote the total amount of data generated during the on period. Then $S = PT_{on} = \sigma\frac{P}{P-\rho}$. S is known as the source *burst-size*.

In order to model the statistical independence of traffic sources, we introduce indeterminate "phases" to the sources as in [5]. Assume that traffic sources are grouped into J classes, and for $1 \leq j \leq J$, there are K_j sources in class j, $\sum_{j=1}^{J} K_j = N$. Each source i of class j, $a_{ji}(t)$, is an extremal on-off, periodic process with leaky bucket parameters (ρ_j, σ_j, P_j), but having an associated phase θ_{ji}, *i.e.*, $a_{ji}(t) = a_j(t + \theta_{ji})$. For $1 \leq j \leq J$ and $1 \leq i \leq K_j$, the phases θ_{ji} are independent random variables uniformly distributed in the interval $[0, T_j]$.

In order to provide robust service, it is imperative to estimate the loss probability P_{loss} at the ATM node due to buffer overflow. However, the loss probability of such a two-resource system with the given on-off sources is very difficult to compute directly. Several bounding and approximate approaches has been developed [6, 10, 11], based on the so-called Benes approach [1]. A new approach is presented in [5] by trading one resource for the other, using the notion of the virtual buffer/trunk.

Based on the work of [5], in this section, we present a new approach to the problem of estimating system loss probability by transforming the two-resource problem into two independent single-resource problems which allows us to explore the optimal trade-off between buffer and bandwidth. This reduction is achieved via a virtual lossless segregate system functioning as a "resource separator", a new interpretation of the notion of the virtual buffer/trunk system introduced in [5].

The rest of this section is organized as follows. In Section 3.1, we introduce the concept of virtual buffer/trunk system. In Section 3.2, we describe our approach for determining the optimal buffer/bandwidth separation and trade-off. The effectiveness of our approach is evaluated via numerical examples in section 4.

3.1 Virtual Buffer/Trunk System

Consider a virtual circuit with a single extremal on-off, periodic traffic source $a(t)$ characterized by parameters (ρ, σ, P). Suppose it is allocated a trunk of bandwidth of c where $\rho \leq c \leq P$. Then the maximum backlog at the virtual circuit is $b = \sigma - T_{on}(c - \rho) = T_{on}(P - c)$. Hence if the virtual circuit is allocated buffer space b, no losses will occur. Following [5], we call such a virtual circuit a *virtual* buffer/trunk system.

Let $u(t)$ and $v(t)$ denote, respectively, the utilized bandwidth and the buffer content of the virtual circuit at time t. As shown in Figure 3, the two processes $u(t)$ and $v(t)$ are periodic with period T, the source period. Let D_{on} denote the time in each cycle that the system is busy, *i.e.*, the buffer is not empty; D_{on} exceeds the length of the source on period by the time required to deplete the buffer.

We can view the virtual buffer/trunk system as a "resource separator" as it splits the traffic process $a(t)$ into two separate processes $u(t)$ and $v(t)$, representing, respectively, the bandwidth requirement and buffer requirement of the source at time t. Under this interpretation, by varying the trunk bandwidth c, the virtual buffer/trunk system can "regulate" the source's buffer and bandwidth requirements, thus providing an interesting buffer/bandwidth trade-off when allocating resources. When c is increased, the source's bandwidth requirement during the system busy period is also increased. However, the system busy period is shortened and the source' buffer requirement is decreased. On the other hand, when c is decreased, the reverse is true.

The effect of varying c on $u(t)$ and $v(t)$ can be concisely stated using the theory of stochastic orderings. The details can be found in [7].

3.2 Estimating Loss Probability for Statistical Service

Recall that we are assuming that the system resources are not sufficient to support lossless service, i.e., (C, B) lies below the buffer/bandwidth trade-off curve in Figure 4. In this section, we present a new approach for estimating the system loss probability in such cases. This approach exploits the buffer/bandwidth separation determined by a virtual buffer/trunk system and exploits the optimal buffer/bandwidth trade-off curve for lossless service.

Consider a lossless segregated system with a total amount of bandwidth C_v and buffer space B_v where (C_v, B_v) lies on the buffer/bandwidth trade-off curve. Because the system resource pair (B, C) lies below the buffer/bandwidth trade-off curve, we must have either $C_v > C$ or $B_v > B$ or both. We call such a system a *virtual* lossless segregated system. In the virtual lossless segregated system, each source i of class j, $a_{ji}(t)$, $1 \le j \le J, 1 \le i \le K_j$, has a trunk of fixed bandwidth c_j^v and a buffer of fixed size b_j^v such that $\sum_{j=1}^{J} K_j c_j^v = C_v$, $\sum_{j=1}^{J} K_j b_j^v = B_v$, and the resources c_j^v and b_j^v are allocated to each virtual buffer/trunk system according to the optimal resource allocation scheme described in Section 2.1. Hence no sources suffer any losses in the virtual buffer/trunk systems.

Let $u_{ji}(t)$ and $v_{ji}(t)$ denote the utilized bandwidth and the buffer contents of source $a_{ji}(t)$ in the virtual buffer/trunk system, where $u_{ji}(t)$ and $v_{ji}(t)$ are two periodic processes synchronized with source $a_{ji}(t)$ (*i.e.*, they all have the same phase θ_{ji}). Interpreting the virtual buffer/trunk system as a "resource separator", then $u_{ji}(t)$ and $v_{ji}(t)$ represent, respectively, the bandwidth and buffer consumed by source $a_{ji}(t)$ at time t. Thus at any time t, the total bandwidth requirement of all sources is $U(t) = \sum_{j=1}^{J} \sum_{i=1}^{K_j} u_{ji}(t)$, while the total buffer requirement of all sources is $V(t) = \sum_{j=1}^{J} \sum_{i=1}^{K_j} v_{ji}(t)$. The virtual lossless segregated system separates the bandwidth and buffer requirements of the sources, thus enabling us to treat them separately.

By imagining that the traffic sources go through a virtual lossless segregated system that separates their bandwidth and buffer requirements, we reduce the difficult task of estimating the system loss probability in a buffered multiplexor with finite resources into that for two simpler systems: a trunk with bandwidth C (but no buffer) and a storage system with buffer space B (but no server). *At any time t, the sources demand a total amount of bandwidth $U(t)$ from the trunk and a total amount of buffer space $V(t)$ from the storage system. The sources will incur losses if either $U(t) > C$ or $V(t) > B$.* Therefore, we can use the probability that either event occurs as an upper bound on the loss probability P_{loss} of the real system. (This buffer/bandwidth separation approach for estimating the system loss probability is justified and made rigorous in [7].) By choosing different resource pairs (C_v, B_v) along the buffer/bandwidth trade-off curve, the virtual lossless segregate system "regulates" the sources' bandwidth and buffer requirements, thus providing a trade-off between them. The rest of this section is devoted to the determination of the resource pair (C_v, B_v) that optimizes the system loss probability estimate.

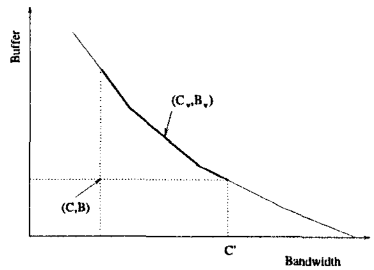

Figure 4: Relationship between (C, B) and the buffer/bandwidth trade-off curve (C_v, B_v).

Let u_{ji} and v_{ji} be two random variables that represent respectively the instantaneous bandwidth requirement and buffer requirement of source a_{ji} at a random time. Then u_{ji} is a Bernoulli random variable taking value c_j^v with probability $w_j = \frac{\rho_j}{c_j^v}$ and 0 with probability $1 - w_j$, and v_{ji} has the distribution $Pr\{v_{ji} \le x\} = 1 - w_j + w_j \frac{x}{b_j^v}$, $0 \le x \le b_j^v$. Moreover, u_{ji}, $1 \le j \le J, 1 \le i \le K_j$, are all independent, as are v_{ji}, $1 \le j \le J, 1 \le i \le K_j$.

Define $U = \sum_{j=1}^{J} \sum_{i=1}^{K_j} u_{ji}$ and $V = \sum_{j=1}^{J} \sum_{i=1}^{K_j} v_{ji}$. From the above discussion it follows that

$$P_{loss} \le \Pr\{V > B \text{ or } U > C\} \le \Pr\{V > B\} + \Pr\{U > C\}. \quad (8)$$

Since (8) is valid for any choice of a resource pair (C_v, B_v) on the buffer/bandwidth trade-off curve, we have

$$P_{loss} \le \inf_{(C_v, B_v)} \{\Pr\{U > C\} + \Pr\{V > B\}\}. \quad (9)$$

The minimization problem on the right hand side of (9) reveals an interesting trade-off between buffer and bandwidth in the virtual buffer/trunk systems. Intuitively, the resource pair (C_v^*, B_v^*) that minimizes the probability on the right hand side of (9) represents the "best" separation of bandwidth and buffer requirements of the resources. This separation is obtained by optimizing the resource allocation along the optimal buffer/bandwidth trade-off curve.

For any C_v, $\Pr\{U > C\}$ and $\Pr\{V > B\}$ can be estimated using the Chernoff bound. For $1 \le j \le J$, let $M_{u_j}(\theta) = E[e^{\theta u_{ji}}]$ and $M_{v_j}(\theta) = E[e^{\theta v_{ji}}]$ denote the moment generating functions of u_{ji} and v_{ji}.

Define $\Lambda_{C_v}(\theta) = \sum_{j=1}^{J} \kappa_j \log M_{u_j}(\theta)$ and $\Lambda_{B_v}(\theta) = \sum_{j=1}^{J} \kappa_j \log M_{v_j}(\theta)$ where $\kappa_j = \frac{K_j}{N}$, $1 \le j \le J$, and $\sum_{j=1}^{J} \kappa_j = 1$. Then Chernoff bound yields $\Pr\{U > C\} \le e^{-N\Lambda^*_{C_v}(C/N)}$ and $\Pr\{V > B\} \le e^{-N\Lambda^*_{B_v}(B/N)}$ where $\Lambda^*_{C_v}(\alpha) = \sup_{\theta \ge 0}\{\theta\alpha - \Lambda_{C_v}(\theta)\}$, and $\Lambda^*_{B_v}(\alpha) = \sup_{\theta \ge 0}\{\theta\alpha - \Lambda_{B_v}(\theta)\}$. Hence,

$$P_{loss} \le \inf_{(C_v, B_v)} \left\{ e^{-N\Lambda^*_{C_v}(C/N)} + e^{-N\Lambda^*_{B_v}(B/N)} \right\}. \quad (10)$$

The optimization problem can be greatly simplified by considering the asymptotic scaling $N \to \infty$ with $C/N = \bar{c}$, $B/N = \bar{b}$ and $K_j/N = \kappa_j$, $1 \le j \le J$, held constant. In this case, we have that

$$\lim_{N \to \infty} \frac{1}{N} \log P_{loss} \le - \sup_{(C_v, B_v)} \left\{ \min\{\Lambda^*_{C_v}(\bar{c}), \Lambda^*_{B_v}(\bar{b})\} \right\}. \quad (11)$$

Since $\Lambda^*_{C_v}(\bar{c})$ is a decreasing function in C_v while $\Lambda^*_{B_v}(\bar{b})$ is an increasing in C_v, the above optimization can be solved in a straightforward manner [7]. Let C_v^* denote the solution to the optimization problem, then the loss probability P_{loss} can be estimated by $e^{-N\Lambda^*_{C_v^*}(\bar{c})} + e^{-N\Lambda^*_{B_v^*}(\bar{b})}$. This estimate can be further refined by adding a prefactor that represents an asymptotic correction term for the Chernoff bound [2].

4 Numerical Examples

In this section, we present numerical examples to illustrate the results of the previous sections. Our focus is on the relationship between source time scale and the optimal buffer/bandwidth trade-off, and on the effect of this buffer/bandwidth trade-off on admissible regions under both deterministic lossless service and statistical service with small loss probabilities. In comparison to previous works that consider multiplexing of periodic on-off sources [5, 6, 9, 10, 11], an important contribution of our present work lies in exploiting the different sources times scales existing in heterogeneous traffic sources. In particular, we show how the buffer/bandwidth trade-off is determined by the source time scale separation and that the boundary of the admissible region for heterogeneous sources can be severely non linear if the sources have very different time scales.

In the following examples, we describe regulated sources using the token rate ρ, the peak rate P, and the burst-size S in place of the usual token bucket size σ. σ can be obtained from S from the identity $S = \sigma \frac{P}{P-\rho}$. The burst-size S is preferred because it is related to the source time scale T_{on} via $T_{on} = S/P$.

We begin by illustrating the relationship between source time scale and the optimal buffer/bandwidth trade-off. We first consider the case where there is a single class of sources. Under lossless multiplexing, for a multiplexor of bandwidth C with K homogeneous sources, from (2.2), we have that the minimal buffer requirement is given by $B_{min} = T_{on}(KP - C)$. With K fixed, the buffer requirement decreases linearly with the bandwidth and is proportional to the source time scale. At $C = KP$, it becomes zero. The source time scale also determines the rate of

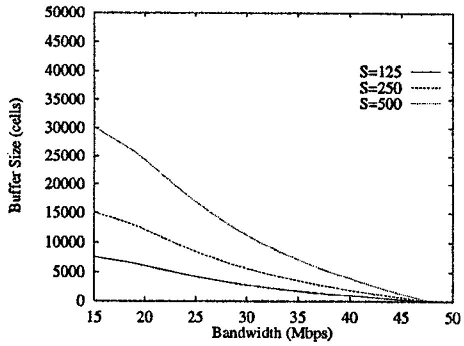

Figure 5: Buffer/bandwidth trade-off for a single class.

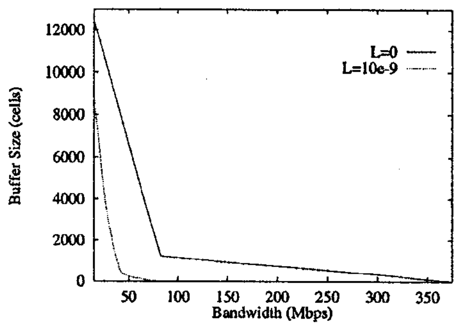

Figure 6: Buffer/bandwidth trade-off for two classes.

change in the buffer requirement, as $dB_{min}/dC = -T_{on}$. In Figure 5, we plot the bandwidth/buffer trade-off for statistical service with a loss probability of $L = 10^{-9}$, where for each given bandwidth value, the buffer requirement is computed as the minimum buffer size such that $P_{loss} \le L$. The curve is plotted with $K = 100$ for three types of single-class sources with the same mean rate $\rho = 0.15$ Mbps and peak rate $P = 1.5$ Mbps, but three different burst-sizes $S = 125, 250, 500$, measured in cells (*i.e.*, 53 Bytes), yielding $T_{on} = 35, 70, 140$ ms, respectively. With $K = 100$, the aggregate average rate is 15 Mbps and the aggregate peak rate 150 Mbps. In this case, the buffer requirement also decreases with the bandwidth, and is proportional to the source time scale. Note that the buffer requirement under statistical service is much less than under lossless service. For example, in this case, the buffer requirement drops to 0 as bandwidth approach 50 Mbps, as opposed to 150 Mbps in the case of lossless service. The presence of statistical multiplexing gains is clearly evident.

We now consider multiplexing two classes of sources. The parameters for the two classes are listed in Table 1. The parameters are chosen so that the two classes have drastically different T_{on} ($T_{on_1} \gg T_{on_2}$), so as to highlight the effect of source time scale on the buffer/bandwidth trade-off curve when heterogeneous classes are multiplexed. We assume that $K_1 = K_2 = 50$. The aggregate average and peak rate of the two classes are 15 Mbps and 375 Mbps, respectively.

The optimal buffer/bandwidth trade-off curve for loss-

class	ρ	P	S	T_{on}
1	0.15	1.5	250	70
2	0.15	6	25	1.7

Table 1: Sources leaky bucket parameters.

less service ($L = 0$) is plotted in Figure 6(a). We can distinguish two distinct components in the curve: the first one with a very steep slope, and the second one with a much lower slope. The knee point is at $C = 82.5$. This phenomenon can be explained by considering the structure of the optimal resource allocation scheme. With small values of bandwidth C, class 2 sources have a "fast" time scale (the time scale index k defined in (3) is 1), and are thus allocated an amount of bandwidth equal to their mean rate ρ_2 and buffer space equal to their maximum token bucket size σ_2, while the remaining bandwidth and buffer space are allocated among class 1 sources, which have much larger time scale, T_{on_1}. As C increases, the bandwidth allocated to class 1 sources increases, hence their buffer requirement decreases as a rate of $dB_{min}/dC = -T_{on_1}$. When C reaches 82.5 Mbps, class 1 sources are allocated peak rate (the time scale index, k, is 2), thus turning into "slow" time scale sources with no buffer requirements. As a consequence, as C further increases, additional bandwidth is allocated only among class 2 sources, reducing their buffer requirements at a rate of $dB_{min}/dC = -T_{on_2}$.

In Figure 6, we also plot the buffer/bandwidth trade-off for statistical service with a loss probability of 10^{-9}. As expected, both bandwidth and buffer requirements are reduced under statistical service, again providing evidence of statistical multiplexing gains. At $C = 85$ Mbps, the buffer requirement drops to 0. For statistical service, the time scale separation of the two classes is easier to discern by examining the trade-off curve in log-scale (Figure 6). Observe that as the bandwidth C approaches 50Mbps, the curve bends. As in the deterministic case, this phenomenon can be explained in terms of the structure of the optimal resource allocation for the virtual segregated system, which is determined by the optimization in (11). Due to statistical multiplexing gains, the time scale separation evident in the bandwidth/buffer trade-off curve occurs at a lower value of bandwidth.

Next, we study the impact of source characteristics on the system admissible region, defined as the number of sources that can be admitted without violating a given QoS requirement. We consider the same two classes of traffic discussed above. We fix the value of the system bandwidth C at 45 Mbps and study the admissible region as a function of the system buffer size B. Because the two classes have the same average rate, the system utilization is maximized when the number of sources of both classes, $K_1 + K_2$ is maximized. Because $\rho/C = 1/300$, the utilization is $\frac{K_1+K_2}{3}\%$.

The admissible regions for lossless service with various values of buffer size are plotted in Figure 7. The key observation here is the non-linearity of the boundary of the admissible region for all values of the buffer size B except for $B = 0$. As we will see, this non-linearity is caused

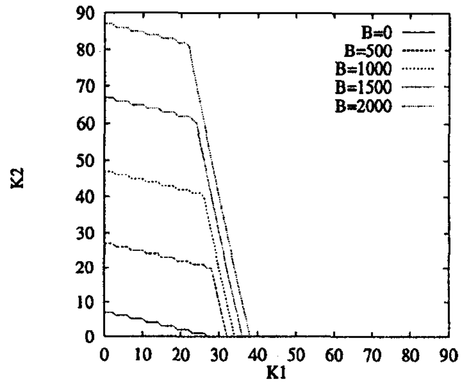

Figure 7: Admissible region for two heterogeneous classes: $T_{on_1} \gg T_{on_2}$.

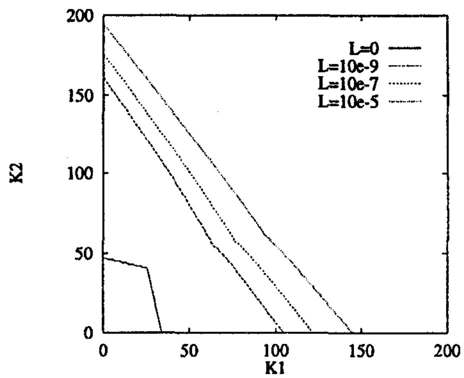

Figure 8: Admissible region: $B = 1000$.

by the different source time scales, T_{on}, of the sources. For $B = 0$, the boundary is linear with a slope given by the ratio of the source peak rates $dK_2/dK_1 = -P_1/P_2$. This is because in the bufferless case, sources are allocated peak rate. For a nonzero buffer size, we can identify two distinct segments of different slopes on the boundary of the admissible region. For relatively small numbers of class 1 sources, the boundary has the same slope as in the bufferless case, as shown in the figure. On the other hand, for relatively small numbers of class 2 sources, it is not difficult to show that the slope of the boundary is

$$\frac{dK_2}{dK_1} = -\frac{P_1}{\sigma_2/T_{on_1} + \rho_2} \leq -\frac{P_1}{\sigma_2/T_{on_2} + \rho_2} = -\frac{P_1}{P_2} \quad (12)$$

as $T_{on_1} \geq T_{on_2}$. As a result, the admissible region is convex.

In Figures 8 and 9, we plot the admissible regions for statistical service with various loss probabilities. The system bandwidth is again fixed at $C = 45$ Mbps, and two values of the system buffer size are considered: $B \in \{1000, 10000\}$ cells. Note that statistical service provides considerable improvement in system utilization. It is also interesting to observe that for large buffer, a form of system saturation takes effect that limits statistical multiplexing gains. For example, for $B = 10000$ (Figure 9), because the most

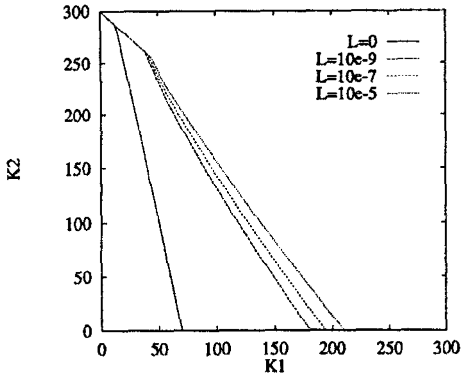

Figure 9: Admissible region: $B = 10000$.

resource-efficient way to accommodate "fast" time scale is to allocate minimum bandwidth and maximum buffer space, when the input traffic is dominated by the "fast" time scale class 2 sources, it is possible to admit up to 300 sources, corresponding to 100% utilization under lossless service. The same buffer size is less effective when input traffic is dominated by the "slow" time scale class 1 sources. In this case, because the most resource-efficient way to accommodate "slow" time scale is to allocate maximum bandwidth and no buffer space, no more than 70 sources can be admitted under lossless service (resulting in an utilization below 25%) and no more than approximately 210 sources can be admitted under statistical service with a loss probability of 10^{-5} (resulting in an utilization of approximately 70%). For statistical service, the resulting admissible region is also clearly non-linear. We can distinguish two distinct components in the boundary: a linear region for $K_1 \leq 45$ and a concave region otherwise.

5 Conclusions

In this paper, we studied the problem of resource allocation and control for an ATM node with regulated traffic. Both guaranteed lossless service and statistical service with small loss probability were considered. We investigated the relationship between source characteristics and buffer/bandwidth trade-off under both services.

For guaranteed lossless service, we identified the optimal resource allocation scheme for an ATM node with finite bandwidth and buffer space, and found that the optimal resource allocation scheme suggests an interesting time scale separation of sources sharing the ATM node. With respect to this time scale separation, the time scale of a source can be defined and the optimal buffer/bandwidth trade-off determined by the sources' time scale. For statistical service with small loss probability, we presented a new approach for estimating loss probability in a shared buffer multiplexor with extremal on-off, periodic sources. Under this approach, the optimal resource allocation for statistical service is achieved by maximizing both the benefits of buffering sharing and bandwidth sharing. The optimal buffer/bandwidth trade-off is again determined by the time scale separation and reflects the efficacy of buffer sharing and bandwidth sharing among sources with different time scales. Through numerical investigations, we illustrated the relationship of source time scale and the optimal buffer/bandwidth tradeoff and discussed the implications of our results in resource allocation and call admission control.

Our results have many other implications in network design and control such as network dimensioning and traffic shaping, in addition to resource allocation and call admission control in an ATM node. This will be the subject of future research.

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