Source Time Scale and Optimal Buffer/Bandwidth Tradeoff for Heterogeneous Regulated Traffic in a Network Node

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***Abstract*— In this paper, we study the problem of resource allocation and control for a network node with regulated traffic. Both guaranteed lossless service and statistical service with small loss probability are considered. We investigate the relationship between source characteristics and the buffer/bandwidth tradeoff under both services. Our contributions are the following. For guaranteed lossless service, we find that the optimal resource allocation scheme suggests that sources sharing a network node with finite bandwidth and buffer space divide into groups according to time scales defined by their leaky-bucket parameters. This time-scale separation determines the manner by which the buffer and bandwidth resources at the network node are shared among the sources. For statistical service with a small loss probability, we present a new approach for estimating the loss probability in a shared buffer multiplexor using the "extremal" ON–OFF, periodic sources. Under this approach, the optimal resource allocation for statistical service is achieved by maximizing both the benefits of buffering sharing and bandwidth sharing. The optimal buffer/bandwidth tradeoff is again determined by a time-scale separation.**

***Index Terms*— Call admission control, network dimensioning, quality-of-service guarantees, resource allocation, statistical multiplexing.**

I. Introduction

RESOURCE allocation is an extremely challenging and important problem in the design and control of high-speed networks, such as asynchronous transfer mode (ATM) networks and the future Integrated Services Internet. The problem is particularly complicated by the need to support *quality-of-service* (QoS) guarantees for a variety of applications with very diverse traffic characteristics.

In [10], a new approach to resource allocation in an ATM node with fixed bandwidth and a finite shared buffer is presented. In this approach, an ATM node is modeled by a shared buffer multiplexor fed by "extremal" ON–OFF periodic arrival processes, which account for the worst-case stochastic behavior (as proved for the bufferless multiplexor in [14], [20]). The ingenuity of the approach is the reduction of the two-resource (i.e., buffer and bandwidth) allocation problem to a single-resource allocation problem, i.e., the known problem of estimating loss probability of a bufferless multiplexor. This reduction is made possible by introducing the concept of a virtual buffer/trunk system and establishing the "exchangeability" of buffer and bandwidth. Based on the analytical results, a qualitative theory is then described which provides many insights in call admission control.

Motivated and inspired by the work in [10], we study source characteristics and their impact on buffer/bandwidth tradeoff in the design and control of a network node. The starting point of our approach is the examination of optimal resource allocation schemes for guaranteed lossless service. We find that under such service, the optimal resource allocation scheme for a network node where each connection has its own allocated bandwidth and buffer space with no resource sharing (referred to as a lossless segregated system) is no different from that for a network node with all connections sharing the resources (i.e., a lossless multiplexing system). Hence, in this case, there are no benefits in resource sharing, and the two systems are effectively equivalent. We also find that the optimal resource allocation scheme suggests an interesting separation of time scales among sources sharing a network node with finite bandwidth and buffer space, with the optimal buffer/bandwidth tradeoff being determined by this time-scale separation. Sources are classified as having either "fast" or "slow" time scales, reflecting the efficacy of either buffer sharing or bandwidth sharing among the sources.

For statistical service where a small loss probability is allowed, we derive a new approach for estimating the loss probability using our results for the optimal buffer/bandwidth tradeoff obtained for lossless service. By giving a new interpretation to the virtual trunk/buffer systems introduced in [10], we are able to transform the two-resource allocation problem into two independent single-resource allocation problems. The best buffer/bandwidth separation is explored by optimizing the resource allocation along the optimal buffer/bandwidth tradeoff curve. Through numerical examples, we demonstrate that source time scales also have a major impact on the optimal resource allocation under statistical service, and that the optimal buffer/bandwidth tradeoff is again reflected by the source time-scale separation.

Our work differs from [10] in several aspects. First, our perspectives on resource allocation and control problems are

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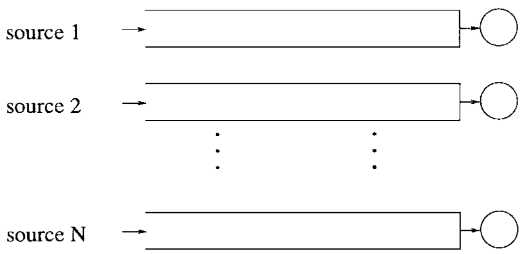

Fig. 1. Lossless segregated system.

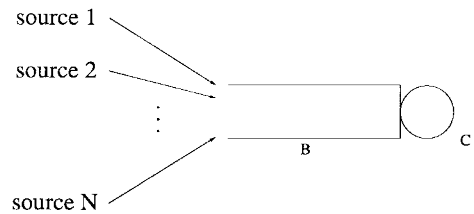

Fig. 2. Lossless multiplexing system.

somewhat different. The authors in [10] are primarily interested in call admission control. This is reflected in their fixing the system bandwidth C and buffer space B. In our approach, we fix one resource (bandwidth C) and find the optimal allocation of the other resource (buffer space B). Furthermore, resource allocation is made according to the time-scale separation of the system and the source' own time scale. Due to this difference in perspectives, we are able to study the role of source time scale and investigate optimal buffer/bandwidth tradeoff for both lossless service and statistical service with a given loss probability. We are also able to explore the maximal benefits of both buffer sharing and bandwidth sharing. This is important, as the efficacy of buffer sharing and that of bandwidth sharing for sources with different time scales are quite different. Numerical examples indicate that our approach provides a better estimate of the system loss probability than that of [10].

The remainder of this paper is organized as follows. We start with the optimal resource allocation problem for guaranteed lossless service in Section II. In Section III, we study the optimal resource allocation problem for statistical service with small loss probability and present a new approach to estimate the system loss probability by maximizing the efficacy of buffer and bandwidth sharing. In Section IV, numerical examples are presented to illustrate the effectiveness of our approach. The paper is concluded in Section V.

II. Guaranteed Lossless Service

The starting point of our study is the analysis of the optimal resource allocation scheme for guaranteed lossless service. Consider a network node with a total amount of bandwidth C and buffer space B. Suppose there are N connections sharing the node. Each connection is associated with a traffic source that is leaky bucket regulated [1], [17]. We consider the following two scenarios. In the first, each connection is allocated a fixed portion of the total bandwidth and buffer space with no resource sharing among the connections. We call this a *lossless segregated system* (see Fig. 1). In the second scenario, the resources are shared among the connections. We call such a system a *lossless multiplexing system* (see Fig. 2). We are interested in optimal resource allocation schemes that, for given bandwidth C, minimize the buffer requirement while ensuring that no connections ever incur losses in the above scenarios. Because of resource sharing, one may expect that the latter system requires less resources than the former for supporting lossless service. However, we show that for guaranteed lossless service, the optimal resource allocation schemes for both systems are the same. Hence, in terms of resource requirements, the lossless multiplexing system is effectively equivalent to the lossless segregated system, and resource sharing in this case does not yield any resource savings. Before we present the optimal resource allocation problems for the two systems, we first describe the regulated traffic sources.

A leaky-bucket regulator is characterized by three parameters: the token rate ρ, the token bucket size σ, and the peak rate P, where $P \geq \rho$. Let $A[\tau, \tau+t]$ denote the amount of traffic passing through the regulator in the time interval $[\tau, \tau+t]$, then

$$A[\tau, \tau+t] \leq \mathcal{E}(t) := \min\{Pt, \sigma+\rho t\}, \qquad \tau \geq 0, t \geq 0 \tag{1}$$

where $\mathcal{E}(t)$ is called the minimum envelope process for the regulated source [5]. It bounds the amount of traffic departing from the regulator during any time interval of length t. For an arbitrary arrival A of a source such that (1) holds, we say that the arrival process A of the source *conforms* to the minimum envelope process $\mathcal{E}$, denoted by $A \sim \mathcal{E}$ [7].

Let T_{on} denote the maximum length of a peak rate burst, i.e.,

$$t_{\text{on}} = \frac{\sigma}{P-\rho}. \tag{2}$$

If a traffic source generates traffic at peak rate P for t_{on} time and switches to rate ρ for the rest of the time, then $A[0,t] = \mathcal{E}(t)$. We call such a traffic source *greedy*.

For the purpose of exposition, we assume that the N traffic sources are classified into J classes according to their regulator characterization, where all regulated sources in class j have the same leaky bucket parameters (ρ_j, σ_j, P_j), $1 \leq j \leq J$. There are K_j class-j sources, and $\Sigma_{j=1}^{J} K_j = N$. We assume that all classes have different peak rate burst length T_{on}. Without loss of generality, let $T_{\text{on}_1} > T_{\text{on}_2} > \cdots > T_{\text{on}_J}$. In order to have a stable system, we require that $\Sigma_{j=1}^{J} K_j\rho_j \leq C$. Also in order to avoid triviality, we assume that $C \leq \Sigma_{j=1}^{J} K_j P_j$.

A. Lossless Segregated System

We first consider the optimal resource allocation problem for the lossless segregated system (Fig. 1). We fix the total bandwidth C for the system, and consider allocation schemes that minimize the total buffer space required to ensure that no connections incur any losses.

For $j = 1, \cdots, J$, suppose each source in class j is allocated bandwidth c_j and buffer space b_j. Let $C_j = K_j c_j$ and $B_j = K_j b_j$ denote, respectively, the total amount of bandwidth and buffer space allocated to class-j sources. The stability condition requires that $c_j \geq \rho_j$ for each j. Since the total bandwidth of the system is C, $\Sigma_{j=1}^{J} K_j c_j \leq C$. For a queue with a server of service capacity c_j, it is well known (see, e.g., [7]) that the queue length at time t, $Q_j(t)$, satisfies the following relation:

$$Q_j(t) = \sup_{0 \le \tau \le t} \{A_j[\tau, t] - c_j(t - \tau)\}$$

where A_j denotes an arbitrary sample path of the arrival process of a source in class j. Since each source in class j is (ρ, σ, P) leaky-bucket regulated, from (1), we have $A_j \sim \mathcal{E}_j$, i.e., $A_j[\tau, \tau + t] \le \mathcal{E}_j(t)$ for any $\tau \ge 0$ and $t \ge 0$. Therefore, the maximum queue length for a source in class j, $Q_{j,\max} \le \sup_{t \ge 0} \{\mathcal{E}_j(t) - c_j t\}$ where the equality is attained if the source is greedy, i.e., $A_j[0, t] = \mathcal{E}_j(t)$. Hence, the amount of buffer space b_j allocated to a source in class j to guarantee no loss is given by

$$b_j = Q_{j,\max} = \sup_{t \ge 0} \{\mathcal{E}_j(t) - c_j t\} = \sigma_j - T_{\text{on}_j}(c_j - \rho_j). \quad (3)$$

The overall buffer space required to ensure that no connections encounter losses is thus $B_{\text{seg}} = \Sigma_{j=1}^J K_j b_j$. This determines the buffer requirement under the segregated allocation scheme.

Observe that (3) is linear in c_j, hence the optimal buffer allocation problem can be formulated as the following Linear Programming (LP) problem with c_j as control variables:

$$\begin{aligned} \text{Minimize} \quad & \sum_{j=1}^J K_j \sigma_j - \sum_{j=1}^J K_j T_{\text{on}_j}(c_j - \rho_j) \\ \text{subject to} \quad & \sum_{j=1}^J K_j c_j \le C, \\ & \rho_j \le c_j \le P_j, \qquad j = 1, \cdots, J, \end{aligned}$$

where buffer allocation for each source j is given by $b_j = \sigma_j - T_{\text{on}_j}(c_j - \rho_j)$, $j = 1, \cdots, J$. The first term in the objective function denotes the buffer requirement if the total bandwidth equals the aggregate average rate of the sources, $\Sigma_{j=1}^J K_j \rho_j$, and the second term accounts for the buffer space reduction due to the fact that C exceeds the aggregate average rate. By removing the first term and $K_j T_{\text{on}_j} \rho_j$ from the second term (both of which are constant) from the objective function and reversing its sign, we can rewrite the optimization problem with a new objective function $\Sigma_{j=1}^J K_j T_{\text{on}_j} c_j$, which is to be maximized subject to the same set of constraints.

It is clear that the new objective function increases whenever bandwidth is taken from classes with smaller T_{on} and is allocated to classes with larger T_{on}. As a consequence, the optimal allocation scheme consists of allocating peak rate to as many classes with large T_{on} as possible without violating the constraint $\Sigma_{j=1}^J C_j \le C$, while allocating only the average rate to classes with small T_{on}. Formally, let k be the smallest index, such that

$$\sum_{j=1}^{k-1} K_j P_j + \sum_{j=k}^J K_j \rho_j \le C < \sum_{j=1}^k K_j P_j + \sum_{j=k+1}^J K_j \rho_j. \quad (4)$$

The optimal resource allocation scheme that results in the minimum buffer requirement for the given bandwidth C is as follows:

$$c_j = \begin{cases} P_j, & j = 1, \cdots, k-1 \\ \dfrac{C - \sum_{l \ne k} K_l c_l}{K_j}, & j = k \\ \rho_j, & j = k+1, \cdots, J \end{cases} \quad (5)$$

and

$$b_j = \begin{cases} 0, & j = 1, \cdots, k-1 \\ \sigma_j - T_{\text{on}_j}(c_j - \rho_j), & j = k \\ \sigma_j, & j = k+1, \cdots, J. \end{cases} \quad (6)$$

Note that $\Sigma_{j=1}^J C_j = \Sigma_{j=1}^J K_j c_j = C$, and the buffer requirement B_{seg} has the following closed-form expression in terms of the regulated source parameters

$$B_{\text{seg}} = \sum_{j=k}^N K_j \sigma_j - K_k T_{\text{on}_k}(c_k - \rho_k). \quad (7)$$

B. Lossless Multiplexing System

We now consider the optimal resource allocation problem for the lossless multiplexing system. Again we fix the total bandwidth of the system which is shared by all connections, and determine the minimal buffer space required to guarantee no losses. This shared buffer multiplexing system is equivalent to a shared single queue serviced by a server of capacity C. Let $Q(t)$ denote the queue length of the shared queue at time t, where the arrival process to the queue is the aggregate of the regulated traffic sources defined earlier. Hence

$$Q(t) = \sup_{0 \le \tau \le t} \left\{ \sum_{j=1}^J \sum_{i=1}^{K_j} A_{\text{ji}}[\tau, t] - C(t - \tau) \right\} \quad (8)$$

where A_{ji} denotes an arbitrary sample path of the arrival process of source i in class j, $1 \le i \le K_j$, $1 \le j \le J$. Since each source i in class j is (ρ, σ, P)-leaky bucket regulated, we have $A_{\text{ji}} \sim \mathcal{E}_j$, i.e., $A_{\text{ji}}[\tau, \tau + t] \le \mathcal{E}_j(t)$ for any $\tau \ge 0$ and $t \ge 0$.

Let $\mathcal{E}(t)$ denote the minimum envelope process of the aggregate arrival process of the leaky-bucket regulated sources. It is not too hard to see that $\mathcal{E}(t) = \Sigma_{j=1}^J K_j \mathcal{E}_j(t)$. Let $Q_{\max}$ be the maximum queue length of the shared buffer system. It can be shown that

$$Q_{\max} = \sup_{t \ge 0} \left\{ \sum_{j=1}^J K_j \mathcal{E}_j(t) - Ct \right\}. \quad (9)$$

Hence, in order to ensure that no losses occur in the system, a minimum buffer of size $B_{\text{mux}} = Q_{\max}$ is required.

We now proceed to derive a closed-form expression for B_{mux} in terms of the parameters of the regulated sources. Note that, since $\mathcal{E}_j(t)$ is piecewise linear and concave for each j, $F(t) := \Sigma_{j=1}^J K_j \mathcal{E}_j(t) - Ct$. The maximum of $F(t)$ is attained at a point $t_{\max}$ such that the left derivative $(d^- F/dt)(t_{\max}) \ge 0$ and the right derivative $(d^+ F/dt)(t_{\max}) \le 0$. From the expression for $\mathcal{E}_j(t)$, it is easy to verify that $t_{\max} = T_{\text{on}_k}$, where k is exactly as defined in (4). Specifically, k is the smallest index, such that

$$\sum_{j=1}^{k-1} K_jP_j + \sum_{j=k}^{J} K_j\rho_j \le C < \sum_{j=1}^{k} K_jP_j + \sum_{j=k+1}^{J} K_j\rho_j \tag{10}$$

which gives exactly the same index k as in the lossless segregated system. In this case, however, T_{on_k} has the following physical meaning: T_{on_k} is the time required for the system to reach its maximum queue length when all sources are greedy.

From (9), we derive the minimum buffer requirement to be

$$B_{\text{mux}} = \sum_{j=k}^{J} K_j\sigma_j - T_{\text{on}_k}\left(C - \sum_{j=1}^{k-1} K_jP_j + \sum_{j=k}^{J} K_j\rho_j\right). \tag{11}$$

Comparing (7) and (11) we observe that $B_{\text{seg}} = B_{\text{mux}}$. Hence, the minimum buffer requirement for the lossless multiplexing system is exactly the same as for the lossless segregated system. Thus, we can define $B_{\min} = B_{\text{seg}} = B_{\text{mux}}$ as the buffer requirement for lossless service.

From (10), we observe that if, for $1 \le j \le J$, we define c_j and b_j as in (5) and (6), then $\Sigma_{j=1}^{J} K_jc_j = C$ and $\Sigma_{j=1}^{J} K_jb_j = B_{\min}$. This observation provides the following interesting interpretation as to how the system resources are optimally shared among the regulated sources in the lossless multiplexing system. Specifically, when resources are optimally allocated in the lossless multiplexing system, the connections behave *as if each of them was allocated fixed bandwidth* c_j *and fixed buffer space* b_j, just as in the lossless segregated system. Hence, the lossless multiplexing system can be effectively treated as if it were the lossless segregated system. This observation provides a motivation for the approach we take in Section III for studying resource allocation schemes under statistical multiplexing with small loss probability.

C. Source Time Scale and Optimal Buffer/Bandwidth Tradeoff Curve

Thus far we have studied the resource allocation problem by fixing the bandwidth C. Now we consider the optimal buffer/bandwidth tradeoff for lossless service with a given set of regulated sources.

For any bandwidth C such that $\Sigma_{j=1}^{J} K_j\rho_j \le C \le \Sigma_{j=1}^{J} K_jP_j$, the index k defined in (10) is determined solely by the regulated source parameters and plays a key role in determining the minimal buffer requirement $B_{\min}$ [see (7)]. Hence, in order to study the buffer/bandwidth tradeoff, it suffices to study k as a function of C. From (4), we see that k is nondecreasing in C: $k = 1$ when $C = \Sigma_{j=1}^{J} K_j\rho_j$ and $k = J$ when $C = \Sigma_{j=1}^{J} K_jP_j$. As a consequence, from (5) and (6) we have, for class-j sources, $j = 1, \cdots, J$

$$\frac{dc_j}{dC} \ge 0 \quad \text{and} \quad \frac{db_j}{dC} \le 0. \tag{12}$$

That is, the allocated bandwidth c_j to class-j sources is a nondecreasing function of C, whereas the buffer space b_j allocated to these sources is a nonincreasing function of C. Therefore, the buffer requirement $B_{\min}$ is a decreasing function of C. Moreover, from (5) and (6), we obtain

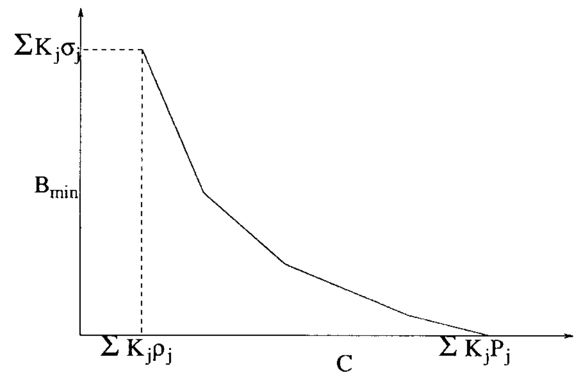

Fig. 3. Optimal buffer/bandwidth tradeoff curve.

$$\frac{dB_{\min}}{dC} = -T_{\text{on}_k}. \tag{13}$$

Thus, the buffer requirement $B_{\min}$ is a piecewise linear decreasing convex function of C (see Fig. 3). We call the curve $(C, B_{\min})$ in Fig. 3 the buffer/bandwidth tradeoff curve for lossless service.

The optimal resource allocation scheme also suggests a taxonomy of regulated sources according to their maximum peak rate burst length T_{on_j}, which we shall also refer to as the time scale of the regulated sources. Subsequently, we call the index k the source *time-scale index* with respect to $(C, B_{\min})$. Sources in class j are said to have either "fast" time scale or "slow" time scale with respect to $(C, B_{\min})$ according to whether $T_{\text{on}_j} < T_{\text{on}_k}$ or $T_{\text{on}_j} > T_{\text{on}_k}$. Under the optimal resource-allocation scheme, we see that the most efficient way to accommodate "fast" time-scale sources under lossless service is to allocate a minimum amount of bandwidth (equal to their mean rates) and a maximum amount of buffer space (equal to their token bucket sizes), while the most efficient way to accommodate "slow" time-scale sources is to allocate maximum bandwidth (equal to their peak rate) and zero buffer space. Sources in class k have intermediate time scales, and accordingly, their buffer/bandwidth tradeoffs are determined by the relation (13). Clearly, with a larger T_{on_k}, increasing the bandwidth allocation to class-k sources will drastically reduce their buffer requirement. Thus, the optimal resource allocation scheme reveals a very interesting relationship between the source time scale and the buffer/bandwidth tradeoff. In Section IV, we will present numerical examples to illustrate this relationship.

III. Statistical Service with Small Loss Probabilities

In the preceding section, we established that, under guaranteed lossless service, the optimal resource allocation is the same, regardless of whether or not resources are shared among sources. In this section, we study the benefits of resource sharing under statistical service where a small probability of loss, say, 10^{-9}–10^{-7} is allowed, and investigate the buffer/bandwidth tradeoff under such statistical service.

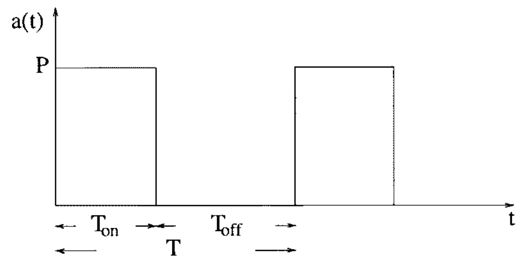

Fig. 4. Extremal ON–OFF periodic curve.

Once again, consider a network node with N connections. Each connection is associated with a leaky bucket regulated traffic source. Suppose the node has a total amount of bandwidth C and buffer space B, shared by all the sources. We assume that the system resources are not sufficient to provide guaranteed lossless service. In other words, the lossless service buffer requirement $B_{\min}$ for the given value of bandwidth C exceeds B. This implies that (C, B) lies below the buffer/bandwidth tradeoff curve in Fig. 3. In this section, we explore the possibility of exploiting statistical multiplexing gains by considering statistical service with small loss probabilities.

Statistical multiplexing gains can be extracted by exploiting the bursty nature and statistical independence of traffic sources [10]. To exploit the bursty nature of traffic sources regulated by leaky buckets with parameters (σ, ρ, P), we follow [10] and assume that the regulated sources are extremal ON–OFF periodic processes after passing through leaky bucket regulators (see Fig. 4). Here, an extremal ON–OFF periodic source is one which, when active, generates data at the peak rate P until the depletion of its token bucket; it then stays inactive until the token bucket is completely filled again. The use of such processes is justified to a large extent by the work of [8], [14], [18]–[20]. In particular, such processes account for the *worst-case* statistical behavior in a bufferless multiplexor in the sense that they maximize the average loss rate [20] and the loss probability estimated by the Chernoff bound [14], [20].

Given a leaky bucket regulator with parameters (ρ, σ, P), define $T_{\text{on}} = (\sigma/P - \rho)$ $T_{\text{off}} = (\sigma/\rho)$ and $T = T_{\text{on}} + T_{\text{off}} = P/\rho T_{\text{on}}$. Here, T_{on} is the maximal time that a (ρ, σ, P) regulated source can send traffic at the peak rate P. This happens when the leaky bucket is full with σ amount of tokens.

T_{off} is the time to fill an empty buffer with the token rate ρ. Consider an ON–OFF periodic source regulated by a (ρ, σ, P) leaky bucket $a(t)$, which periodically becomes active and sends traffic at a rate of P for a period of T_{on}, exhausting all the available tokens; it then becomes inactive for a period of T_{off}, letting the leaky bucket to be filled again. This source is referred to as an *extremal* ON–OFF, periodic departure process from a leaky bucket regulator with parameters (ρ, σ, P) in [10], as it maximizes the loss probability incurred in a bufferless multiplexing system [14]. Let S denote total amount of data generated during an on period time, $S = PT_{\text{on}} = \sigma(P/P - \rho)$. S is known as the source *burst-size*.

In order to model the statistical independence of traffic sources, we introduce indeterminate "phases" to the sources as in [10]. Assume that traffic sources are grouped into J classes, and for $1 \le j \le J$, there are K_j sources in class j, $\Sigma_{j=1}^{J} K_j = N$. Each source i of class j, $a_{\text{ji}}(t)$, is an extremal ON–OFF periodic process with leaky bucket parameters (ρ_j, σ_j, P_j), and has an associated phase θ_{ji}, where $\theta_{\text{ji}} \in [0, T_j]$ and $T_j = T_{\text{on}_j} + T_{\text{off}_j}$. In other words, $a_{\text{ji}}(t) = a_j(t + \theta_{\text{ji}})$. For $1 \le j \le J$ and $1 \le i \le K_j$, the phases θ_{ji} are independent random variables uniformly distributed in the interval $[0, T_j]$.

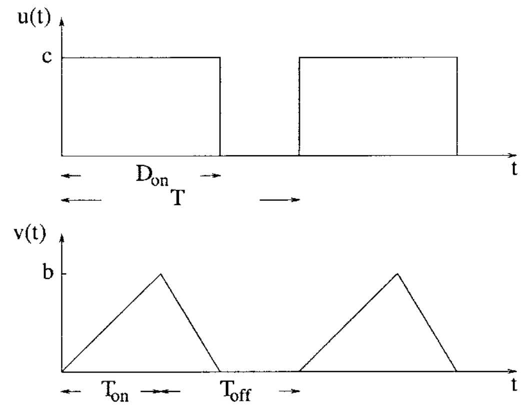

Fig. 5. Buffer/bandwidth separation.

In order to provide robust service, it is imperative to estimate the loss probability P_{loss} at the network node due to buffer overflow. However, the loss probability of such a two-resource system with the given ON–OFF sources is very difficult to compute directly. In this section, we present a new approach to the problem of estimating system loss probability by transforming the two-resource problem into two independent single-resource problems which allows us to explore the optimal tradeoff between buffer and bandwidth. This reduction is achieved via a virtual lossless segregate system functioning as a "resource separator," a new interpretation of the notion of the virtual buffer/trunk system introduced in [10].

A. Virtual Buffer/Trunk System

Consider a connection that consists of a single extremal ON–OFF, periodic traffic source $a(t)$ characterized by parameters (ρ, σ, P). Suppose it is allocated a trunk of bandwidth of c, where $\rho \le c \le P$. Then the maximum backlog at the connection is $b = \sigma - T_{\text{on}}(c - \rho) = T_{\text{on}}(P - c)$. Hence if the connection is allocated buffer space b, no losses will occur. Following [10], we call such a connection with the allocated resources a *virtual* buffer/trunk system.

Let $u(t)$ and $v(t)$ denote, respectively, the utilized bandwidth and the buffer content of the connection at time t. As shown in Fig. 5, the two processes $u(t)$ and $v(t)$ are periodic with period T, the source period. The buffer fills at rate $P - c$ during a source's on period, reaches b at the end of the source on period, and then, at the onset of an off period, depletes at rate c until it empties. Let D_{on} denote the time in each cycle that the system is busy, i.e., the buffer is not empty; D_{on} exceeds the length of the source on period by the time required to deplete the buffer. Thus

$$D_{\text{on}} = T_{\text{on}} + \frac{b}{c}. \tag{14}$$

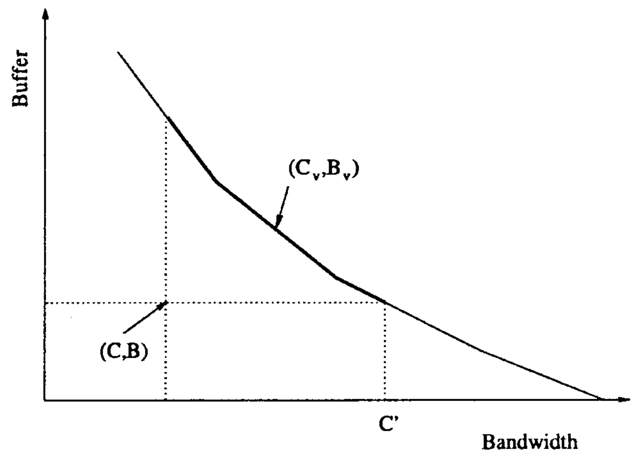

Fig. 6. Relationship between (C, B) and the buffer/bandwidth tradeoff curve (C_v, B_v).

From Fig. 5, the utilized bandwidth $u(t)$ is c whenever the buffer is not empty and 0 otherwise. Similarly, let $v'(t)$ denote the derivative of $v(t)$, i.e., $v(t) = \int_0^t v'(s)\ ds$ for any $t \geq 0$. We see that $v'(t)$ is zero whenever the buffer is empty; otherwise, it takes values $P-c$ and $-c$ during the buffer filling and emptying phase, respectively. Given the definition of $u(t)$ and $v'(t)$, it is easy to verify that for any $t \geq 0$

$$a(t) = u(t) + v'(t). \tag{15}$$

In light of (15), we can view the virtual buffer/trunk system as a "resource separator" which splits the traffic process $a(t)$ into two separate processes $u(t)$ and $v(t)$, representing, respectively, the bandwidth requirement and buffer requirement of the source at time t. Under this interpretation, by varying the trunk bandwidth c, the virtual buffer/trunk system can "regulate" the source's buffer and bandwidth requirements, thus providing an interesting buffer/bandwidth tradeoff when allocating these network resources. When c is increased, the source's bandwidth requirement during the system busy period is also increased. However, the system busy period is shortened and the source's buffer requirement is decreased. When c is increased to the source peak rate P, the system busy period D_{on} equals the source on period T_{on}, during which the bandwidth requirement is P while the buffer requirement is reduced to zero at all times. On the other hand, when c is decreased, the reverse is true. In particular, when c is decreased to the source's average rate ρ, the system is always busy and the buffer is never empty. Thus the source's bandwidth requirement is ρ at all times, and the buffer requirement is uniformly distributed in $[0, \sigma]$. The effect of varying c on $u(t)$ and $v(t)$ can be concisely stated using the theory of stochastic orderings (see [13] for details).

B. Estimating Loss Probability for Statistical Service

Recall that we are assuming that the system resources are not sufficient to support lossless service, i.e., (C, B) lies below the buffer/bandwidth tradeoff curve in Fig. 6. In this section, we present a new approach for estimating the system loss probability in such cases. This approach exploits the buffer/bandwidth separation determined by a virtual

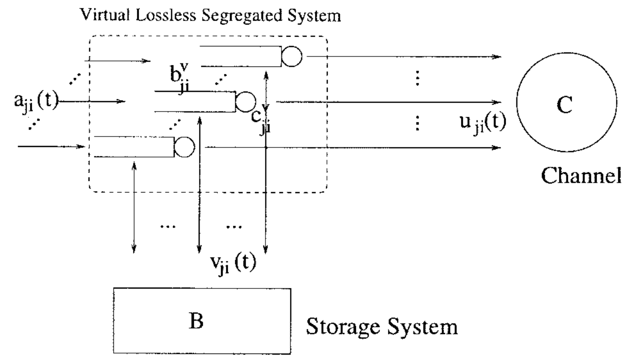

Fig. 7. Buffer/bandwidth separation via virtual lossless segregated system.

buffer/trunk system and exploits the optimal buffer/bandwidth tradeoff curve for lossless service. We model the network node by an infinite queue with a server of capacity C, the arrival process to the queue is the aggregation of the independent extremal ON–OFF, periodic sources $a_{\text{ji}}(t)$, $1 \leq j \leq J$, and $1 \leq i \leq K_j$. The system loss probability P_{loss} can then be upper bounded by the buffer overflow probability $\Pr\{Q > B\}$, where Q denotes the stationary queue length of the infinite queue system.

Now consider a lossless segregated system with a total amount of bandwidth C_v and buffer space B_v where (C_v, B_v) lies on the buffer/bandwidth tradeoff curve. Because the system resource pair (C, B) lies below the buffer/bandwidth tradeoff curve, we must have either $C_v > C$ or $B_v > B$ or both (see Fig. 6). We call such a system a *virtual* lossless segregated system (see Fig. 7). In the virtual lossless segregated system, each source i of class j, $a_{\text{ji}}(t)$, $1 \leq j \leq J, 1 \leq i \leq K_j$, has a trunk of fixed bandwidth c_j^v and a buffer of fixed size b_j^v such that $\Sigma_{j=1}^J K_j c_j^v = C_v$, $\Sigma_{j=1}^J K_j b_j^v = B_v$, and the resources c_j^v and b_j^v are allocated to each virtual buffer/trunk system according to the optimal resource allocation scheme described in Section II-A. Hence, no sources suffer any losses in the virtual buffer/trunk systems.

Let $u_{\text{ji}}(t)$ and $v_{\text{ji}}(t)$ denote the utilized bandwidth and the buffer contents of source $a_{\text{ji}}(t)$ in the virtual buffer/trunk system, where $u_{\text{ji}}(t)$ and $v_{\text{ji}}(t)$ are two periodic processes synchronized with source $a_{\text{ji}}(t)$ (i.e., they all have the same phase θ_{ji}). Interpreting the virtual buffer/trunk system as a "resource separator," then $u_{\text{ji}}(t)$ and $v_{\text{ji}}(t)$ represent, respectively, the bandwidth and buffer consumed by source $a_{\text{ji}}(t)$ at time t. Thus, at any time t, the total bandwidth requirement of all sources is $U(t) = \Sigma_{j=1}^J \Sigma_{i=1}^{K_j} u_{\text{ji}}(t)$, while the total buffer requirement of all sources is $V(t) = \Sigma_{j=1}^J \Sigma_{i=1}^{K_j} v_{\text{ji}}(t)$. The virtual lossless segregated system separates the bandwidth and buffer requirements of the sources, thus enabling us to treat them separately, as illustrated below.

For any $[\tau, t]$, define $A[\tau, t] = \int_\tau^t \Sigma_{j=1}^J \Sigma_{i=1}^{K_j} a_{\text{ji}}(s)\ ds$, $U[\tau, t] = \int_\tau^t \Sigma_{j=1}^J \Sigma_{i=1}^{K_j} u_{\text{ji}}(s)\ ds = \int_\tau^t U(s)\ ds$, and $V[\tau, t] = \int_\tau^t \Sigma_{j=1}^J \Sigma_{i=1}^{K_j} v'_{\text{ji}}(s)\ ds = V(t) - V(\tau)$. From (15), it is clear that $A[\tau, t] = U[\tau, t] + V[\tau, t]$.

Let $Q(t)$ denote the queue length of the infinite queue system at time t. Then it can be shown that

$$Q(t) = \sup_{\tau \leq t}\{A[\tau, t] - C(t-\tau)\} \leq Q_C(t) + Q_B(t) \tag{16}$$

where

$$Q_C(t) = \sup_{\tau \le t} \{U[\tau, t] - C(t - \tau)\}$$

and

$$Q_B(t) = \sup_{\tau \le t} \{V(t) - V(\tau)\} = V(t).$$

The inequality (16) determines the "separation" between bandwidth and buffer requirements that enables us to treat them separately. The key observation now is that $Q_C(t)$ and $Q_B(t)$ can be regarded as the state of two well-defined systems. In particular, $Q_C(t)$ is the queue length of a system with a server of capacity C, where the arrival process is $U(t)$. $Q_B(t) = V(t)$ is the content of a storage system where data is stored and retrieved according to the rate process $V'(t) = dV(t)/dt = \Sigma_{j=1}^{J} \Sigma_{i=1}^{K_j} v'_{\text{ji}}(t)$: data is stored into the system when $V'(t) \ge 0$, and retrieved from the system when $V'(t) < 0$.

Let Q_C and Q_B denote, respectively, the stationary versions of $Q_C(t)$ and $Q_B(t)$ (which exist because of the system stability condition). Inequality (16) implies

$$P_{\text{loss}} \le \Pr\{Q_C > 0\} + \Pr\{Q_B > B\}. \tag{17}$$

For small loss probability and large C, the first term in (17), $\Pr\{Q_C > 0\}$, is well approximated (see [9], [6], [16]) by the loss probability of a *bufferless* multiplexor with capacity C and stationary arrival process U, i.e., $\Pr\{U > C\}$. This, along with the fact that $Q_B = V$, yields the following upper bound on the system loss probability

$$P_{\text{loss}} \le \Pr\{U > C\} + \Pr\{V > B\}. \tag{18}$$

From the above discussion, we see that by visualizing the traffic sources as going through a virtual lossless segregated system that separates their bandwidth and buffer requirements, we reduce the difficult task of estimating the system loss probability in a buffered multiplexor with finite resources to that for two simpler systems: a bufferless channel with bandwidth C and a storage system with buffer space B (see Fig. 7 for an illustration). *At any time t, the sources demand a total amount of bandwidth $U(t)$ from the channel and a total amount of buffer space $V(t)$ from the storage system.* From (18), we see that the loss probability P_{loss} of the real system can be upper bounded by the probability of the events that either $U(t) > C$ or $V(t) > B$.

By choosing different resource pairs (C_v, B_v) along the buffer/bandwidth tradeoff curve, the virtual lossless segregate system "regulates" the sources' bandwidth and buffer requirements, thus providing a tradeoff between them. The rest of this section is devoted to the determination of the resource pair (C_v, B_v) that optimizes the system loss probability estimate.

Since (18) is valid for any choice of a resource pair (C_v, B_v) on the buffer/bandwidth tradeoff curve, we have

$$P_{\text{loss}} \le \inf_{(C_v, B_v)} \{\Pr\{U > C\} + \Pr\{V > B\}\}. \tag{19}$$

The minimization problem on the right hand side of (19) reveals an interesting tradeoff between buffer and bandwidth in the virtual buffer/trunk systems. Intuitively, the resource pair (C_v^*, B_v^*) that minimizes the probability on the right hand side of (19) represents the "best" separation of bandwidth and buffer requirements of the resources. This separation is obtained by optimizing the resource allocation along the optimal buffer/bandwidth tradeoff curve.

For any C_v, $\Pr\{U > C\}$ and $\Pr\{V > B\}$ can be estimated using the Chernoff bound. For $1 \le j \le J$, $1 \le i \le K_j$, let u_{ji} and v_{ji} be two random variables that represent, respectively, the instantaneous bandwidth requirement and buffer requirement of source a_{ji} at a random time. Then u_{ji} is a random variable taking value c_j^v with probability $w_j = (\rho_j / c_j^v)$ and 0 with probability $1 - w_j$, and v_{ji} has the distribution $Pr\{v_{\text{ji}} \le x\} = 1 - w_j + w_j(x/b_j^v)$, $0 \le x \le b_j^v$ (see [13]). Moreover, $\{u_{\text{ji}}\}$ and $\{v_{\text{ji}}\}$, $1 \le j \le J$, $1 \le i \le K_j$, are assumed to be mutually independent. For $1 \le j \le J$, let $M_{u_j}(\theta) = E[e^{\theta u_{\text{ji}}}]$ and $M_{v_j}(\theta) = E[e^{\theta v_{\text{ji}}}]$ denote the moment generating functions of u_{ji} and v_{ji}. Define $U = \Sigma_{j=1}^{J} \Sigma_{i=1}^{K_j} u_{\text{ji}}$ and $V = \Sigma_{j=1}^{J} \sum_{i=1}^{K_j} v_{\text{ji}}$. Then $M_{u_j}(\theta) = E[e^{\theta u_{\text{ji}}}] = 1 - w_j + w_j e^{\theta c_j^v}$ and $M_{v_j}(\theta) = E[e^{\theta v_{\text{ji}}}] = 1 - w_j + (1/b_j^v \theta) w_j (e^{\theta c_j^v} - 1)$, where (c_j^v, b_j^v) is the resource allocation to a source of class j under the optimal resource allocation scheme with total bandwidth C_v. Define $\Lambda_{C_v}(\theta) = \Sigma_{j=1}^{J} \kappa_j \log M_{u_j}(\theta)$ and $\Lambda_{B_v}(\theta) = \Sigma_{j=1}^{J} \kappa_j \log M_{v_j}(\theta)$ where $\kappa_j = (K_j/N)$, $1 \le j \le J$, and $\Sigma_{j=1}^{J} \kappa_j = 1$. The Chernoff bound [3] yields

$$\Pr\{U > C\} = \Pr\left\{\sum_{j=1}^{J} \sum_{i=1}^{K_J} u_{\text{ji}} > C\right\} \le e^{-N\Lambda_{C_v}^*(C/N)} \tag{20}$$

and

$$\Pr\{V > B\} = \Pr\left\{\sum_{j=1}^{J} \sum_{i=1}^{K_J} v_{\text{ji}} > B\right\} \le e^{-N\Lambda_{B_v}^*(B/N)} \tag{21}$$

where

$$\Lambda_{C_v}^*(\alpha) = \sup_{\theta \ge 0} \{\theta\alpha - \Lambda_{C_v}(\theta)\}$$
$$\Lambda_{B_v}^*(\alpha) = \sup_{\theta \ge 0} \{\theta\alpha - \Lambda_{B_v}(\theta)\}. \tag{22}$$

Hence

$$P_{\text{loss}} \le \inf_{(C_v, B_v)} \{e^{-N\Lambda_{C_v}^*(C/N)} + e^{-N\Lambda_{B_v}^*(B/N)}\}. \tag{23}$$

The optimization problem can be greatly simplified by considering the asymptotic scaling $N \to \infty$ with $C/N = \overline{c}$, $B/N = \overline{b}$ and $K_j/N = \kappa_j$, $1 \le j \le J$, held constant. In this case, we have

$$\lim_{N \to \infty} \frac{1}{N} \log P_{\text{loss}} \le -\sup_{(C_v, B_v)} \{\min\{\Lambda_{C_v}^*(\overline{c}), \Lambda_{B_v}^*(\overline{b})\}\}. \tag{24}$$

Actually we can show (see [13]) that the choice of (C_v, B_v) can be restricted to the segment of the buffer/bandwidth tradeoff curve where $C_v \ge C$ and $B_v \ge B$ (the highlighted segment on the buffer/bandwidth tradeoff curve in Fig. 6). Let

$\mathcal{C} = [C, C']$ be the corresponding range of C_v, i.e., $C_v \in \mathcal{C}$ iff $B_v \geq B$ and $C_v \geq C$, where C' is the bandwidth such that the minimum buffer requirement in the lossless segregated system is exactly B. Then we have

$$\lim_{N\to\infty} \frac{1}{N} \log P_{\text{loss}} \leq -\sup_{C_v \in \mathcal{C}} \{\min\{\Lambda^*_{C_v}(\overline{c}), \Lambda^*_{B_v}(\overline{b})\}\}. \tag{25}$$

Since $\Lambda^*_{C_v}(\overline{c})$ is a decreasing function in C_v while $\Lambda^*_{B_v}(\overline{b})$ is an increasing in C_v, the above optimization can be solved in a straightforward manner; details can be found in [13]. Let C_v^* denote the solution to the optimization problem, then the loss probability P_{loss} can be estimated by $e^{-N\Lambda^*_{C_v^*}(\overline{c})} + e^{-N\Lambda^*_{B_v^*}(\overline{b})}$. This estimate can be further refined by adding a prefactor that represents an asymptotic correction term for the Chernoff bound 4. Therefore

$$P_{\text{loss}} \approx \frac{1}{\theta^*_{C_v^*}\sqrt{2\pi\Lambda''_{C_v^*}(\theta^*_{C_v^*})}}\, e^{-N\Lambda^*_{C_v^*}(\overline{c})} + \frac{1}{\theta^*_{B_v^*}\sqrt{2\pi\Lambda''_{B_v^*}(\theta^*_{B_v^*})}}\, e^{-N\Lambda^*_{B_v^*}(\overline{b})} \tag{26}$$

where $\theta^*_{C_v^*}$ and $\theta^*_{B_v^*}$ are the solutions of the equations $\Lambda'_{C_v^*}(\theta) = C$ and $\Lambda''_{B_v^*}(\theta) = B$, respectively and where $\Lambda'(\theta)$ and $\Lambda''(\theta)$ denote the first and second derivatives of $\Lambda(\theta)$.

IV. Numerical Examples

In this section, we present numerical examples to illustrate the results of the previous sections. Our focus is on the relationship between source time scale and the optimal buffer/bandwidth tradeoff, and on the effect of this buffer/bandwidth tradeoff on admissible regions under both deterministic lossless service and statistical service with small loss probabilities. In comparison to previous works that consider multiplexing of periodic ON–OFF sources (see, e.g., [10]–[12], [15]), an important contribution of our present work lies in exploiting the different sources times scales existing in heterogeneous traffic sources. In particular, we show how the buffer/bandwidth tradeoff is determined by the source time-scale separation and that the boundary of the admissible region for heterogeneous sources can be severely non linear if the sources have very different time scales.

In the following examples, we describe regulated sources using the token rate ρ, the peak rate P, and the burst-size S in place of the usual token bucket size σ. σ can be obtained from S from the identity $S = \sigma(P/P - \rho)$. The burst-size S is preferred because it is related to the source time scale T_{on} via $T_{\text{on}} = S/P$.

We begin by illustrating the relationship between source time scale and the optimal buffer/bandwidth tradeoff. We first consider the case where there is a single class of sources. Under lossless multiplexing, for a multiplexor of bandwidth C with K homogeneous sources, from (11), the minimal buffer requirement is given by

$$B_{\min} = T_{\text{on}}(KP - C). \tag{27}$$

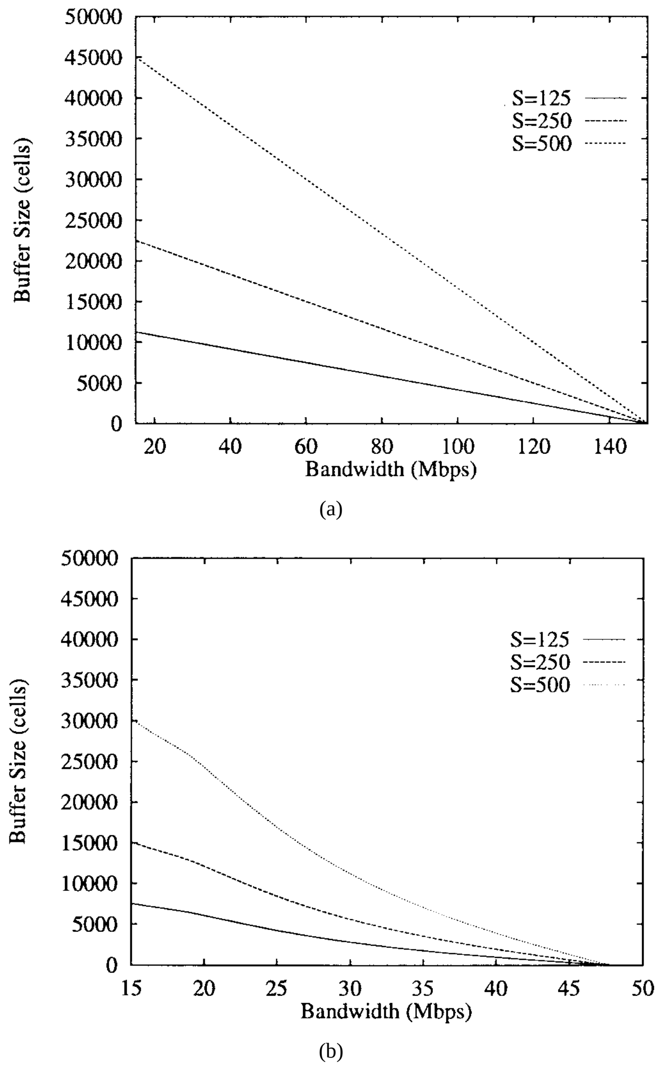

Fig. 8. Buffer/bandwidth tradeoff for a single class $K = 100$. (a) $L = 0$. (b) $L = 10^{-9}$.

With K fixed, the buffer requirement decreases linearly with the bandwidth and is proportional to the source time scale. At $C = KP$, it becomes zero. The source time scale also determines the rate of change in the buffer requirement, as $dB_{\min}/dC = -T_{\text{on}}$. In Fig. 8(a), the optimal buffer/bandwidth curve is plotted with $K = 100$ for three types of single-class sources with the same mean rate $\rho = 0.15$ Mb/s and peak rate $P = 1.5$ Mb/s, but three different burst-sizes $S = 125, 250, 500$, measured in cells (i.e., 53 bytes), yielding $T_{\text{on}} = 35, 70, 140$ ms, respectively. With $K = 100$, the aggregate average rate is 15 Mb/s and the aggregate peak rate 150 Mb/s. In Fig. 8(b), we plot the bandwidth/buffer tradeoff for statistical service with a loss probability of $L = 10^{-9}$ where, for each given bandwidth value, the buffer requirement is computed as the minimum buffer size such that $P_{\text{loss}} \leq L$. In this case, the buffer requirement also decreases with the bandwidth, and is proportional to the source time scale. Note that the buffer requirement under statistical service is much less than under lossless service. For example, in this case, the buffer requirement drops to 0 as bandwidth approach 50 Mb/s,

TABLE I
SOURCES LEAKY-BUCKET PARAMETERS

class	ρ	P	S	T_{on}
1	0.15	1.5	250	70
2	0.15	6	25	1.7

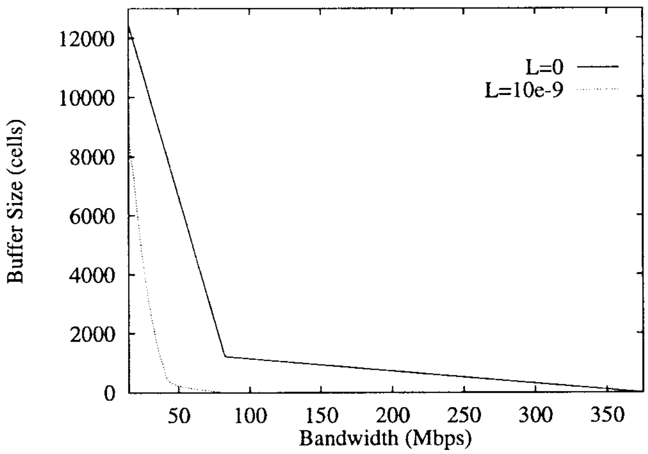

Fig. 9. Buffer/bandwidth tradeoff for two classes.

as opposed to 150 Mb/s in the case of lossless service. The presence of statistical multiplexing gain is clearly evident.

We now consider multiplexing two classes of sources. The parameters for the two classes are listed in Table I. The parameters are chosen so that the two classes have drastically different values of $T_{\rm on}$ ($T_{\rm on_1} \gg T_{\rm on_2}$), so as to highlight the effect of source time scale on the buffer/bandwidth tradeoff curve when heterogeneous classes are multiplexed. We assume that $K_1 = K_2 = 50$. The aggregate average and peak rate of the two classes are 15 and 375 Mb/s, respectively.

The optimal buffer/bandwidth tradeoff curve for lossless service $(L = 0)$ is plotted in Fig. 9. We can distinguish two distinct components in the curve: the first one with a very steep slope, and the second one with a much lower slope with a knee at $C = 82.5$. This phenomenon can be explained by considering the structure of the optimal resource allocation scheme. With small values of bandwidth C, class-2 sources have a "fast" time scale (the time-scale index k defined in (10) is 1), and are thus allocated an amount of bandwidth equal to their mean rate ρ_2 and buffer space equal to their maximum token bucket size σ_2, while the remaining bandwidth and buffer space are allocated among class-1 sources, which have much larger time scale $T_{\rm on_1}$. As C increases, the bandwidth allocated to class-1 sources increases, hence their buffer requirement decreases as a rate of $dB_{\rm min}/dC = -T_{\rm on_1}$. When C reaches 82.5 Mb/s, class-1 sources are allocated their peak rates (the time-scale index k is 2), thus turning into "slow" time-scale sources with no buffer requirements. As a consequence, as C further increases, additional bandwidth is allocated only among class-2 sources, reducing their buffer requirements at a rate of $dB_{\rm min}/dC = -T_{\rm on_2}$.

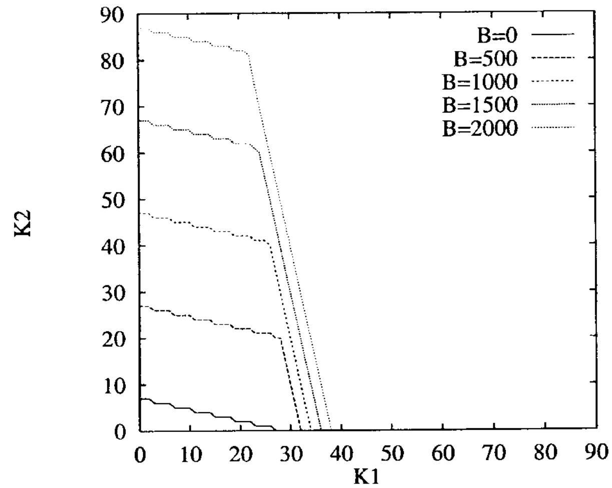

Fig. 10. Admissible region for two heterogeneous classes: $T_{\rm on_1} \gg T_{\rm on_2}$.

Also plotted in Fig. 9 is the buffer/bandwidth tradeoff for statistical service with a loss probability of 10^{-9}. As expected, both bandwidth and buffer requirements are reduced under statistical service, again providing evidence of statistical multiplexing gains. At $C = 85$ Mb/s, the buffer requirement drops to 0. Observe that as the bandwidth C approaches 50 Mb/s, the curve bends. As in the deterministic case, this phenomenon can be explained in terms of the structure of the optimal resource allocation for the virtual segregated system, which is determined by the optimization in (25). Due to statistical multiplexing gains, the time-scale separation evident in the bandwidth/buffer tradeoff curve occurs at a lower value of bandwidth.

Next, we study the impact of source characteristics on the system admissible region, defined as the number of sources that can be admitted without violating a given QoS requirement. We consider the same two classes of traffic discussed above. We fix the value of the system bandwidth C at 45 Mb/s and study the admissible region as a function of the system buffer size B. Because the two classes have the same average rate, the system utilization is maximized when the number of sources of both classes, $K_1 + K_2$ is maximized. Because $\rho/C = 1/300$, the utilization is $(K_1 + K_2/3)\%$.

The admissible regions for lossless service with various values of buffer size are plotted in Fig. 10. The key observation here is the nonlinearity of the boundary of the admissible region for all values of the buffer size B except for $B = 0$. As we will see, this nonlinearity is caused by the different source time scales $T_{\rm on}$ of the sources. For $B = 0$, the boundary is linear with a slope given by the ratio of the source peak rates $dK_2/dK_1 = -P_1/P_2$. This is because in the bufferless case, sources are allocated their peak rates. For a nonzero buffer size, we can identify two distinct segments of different slopes on the boundary of the admissible region. For relatively small numbers of class-1 sources, the boundary has the same slope as in the bufferless case, as shown in the figure. On the other hand, for a relatively small numbers of class-2 sources, it is

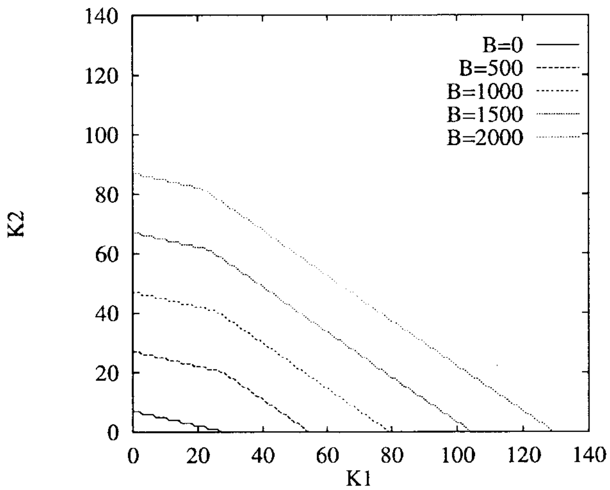

Fig. 11. Admissible region for two heterogeneous classes: $T_{\text{on}_1} \approx T_{\text{on}_2}$.

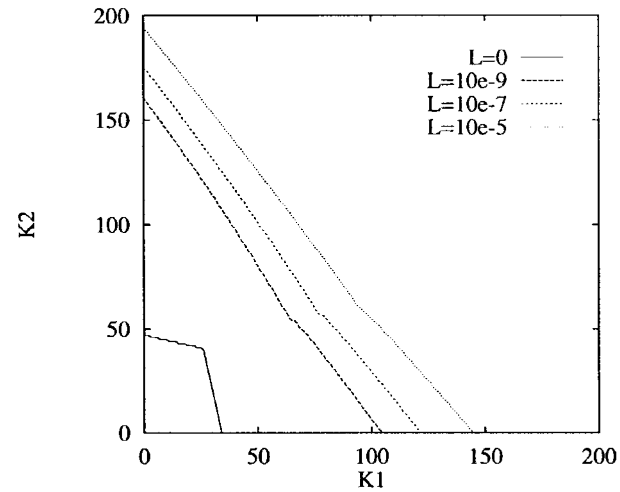

Fig. 13. Admissible region: $B = 1000$.

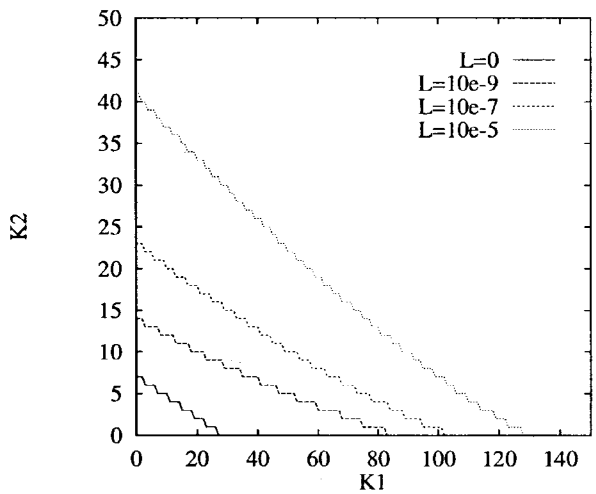

Fig. 12. Admissible region: $B = 0$.

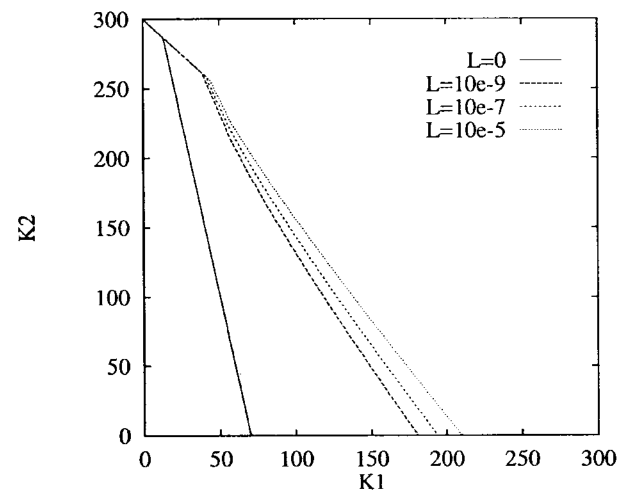

Fig. 14. Admissible region: $B = 10\,000$.

not difficult to show that the slope of the boundary is

$$\frac{dK_2}{dK_1} = -\frac{P_1}{\sigma_2/T_{\text{on}_1} + \rho_2} \leq -\frac{P_1}{\sigma_2/T_{\text{on}_2} + \rho_2} = -\frac{P_1}{P_2} \quad (28)$$

as $T_{\text{on}_1} \geq T_{\text{on}_2}$. As a result, the admissible region is convex.

From (28), we expect the boundary to be approximately linear when $T_{\text{on}_1} \approx T_{\text{on}_2}$, and linear when $T_{\text{on}_1} = T_{\text{on}_2}$. To illustrate this observation, we reduce the burst size S of class 1 from 250 to 20, yielding a new source time scale $T_{\text{on}_1} \approx 5.8$, which is comparable to the time scale of class 2, $T_{\text{on}_2} \approx 3.5$. The resulting admissible region is plotted in Fig. 11. We see that the boundary of the admissible region is now much closer to linear.

In Figs. 12–14, we plot the admissible regions for statistical service with various loss probabilities. The system bandwidth is again fixed at $C = 45$ Mb/s, and three values of the system buffer size are considered: $B \in \{0, 1000, 10\,000\}$ cells. Note that statistical service provides considerable improvement in system utilization. It is also interesting to observe that for large buffer, a form of system saturation takes effect that limits statistical multiplexing gains. For example, for $B = 10\,000$ (Fig. 14), because the most resource-efficient way to accommodate "fast" time scale is to allocate minimum bandwidth and maximum buffer space, when the input traffic is dominated by the "fast" time-scale class-2 sources, it is possible to admit up to 300 sources, corresponding to 100% utilization under lossless service. The same buffer size is less effective when input traffic is dominated by the "slow" time-scale class-1 sources. In this case, because the most resource-efficient way to accommodate "slow" time scale is to allocate maximum bandwidth and no buffer space, no more than 70 sources can be admitted under lossless service (resulting in a utilization below 25%) and no more than approximately 210 sources can be admitted under statistical service with a loss probability of 10^{-5} (resulting in an utilization of approximately 70%). For statistical service, the resulting admissible region is also clearly nonlinear. We can distinguish two distinct components in the boundary: a linear region for $K_1 \leq 45$ and a concave region otherwise.

We now compare our approach with that in [10]. In [10], the system loss probability is estimated by reducing the two-resource allocation problem into a single resource allocation problem by fixing the buffer/bandwidth tradeoff to the ratio B/C regardless of sources characteristics. We extend their

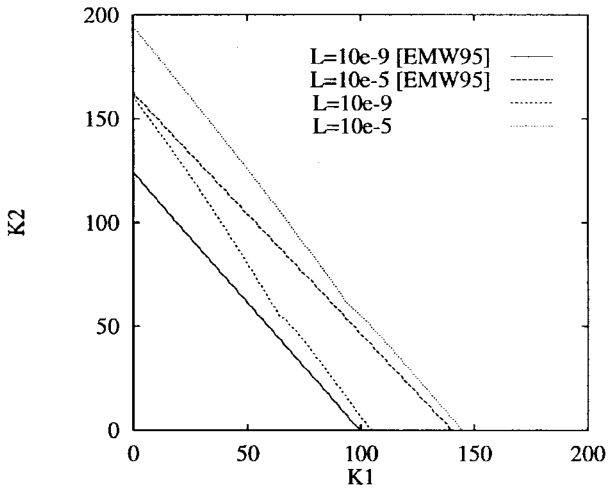

Fig. 15. Comparison of our approach with [EMW95].

TABLE II
COMPARISON WITH SIMULATION

Loss Probability	$2.2 \cdot 10^{-8}$	$1.7 \cdot 10^{-7}$	$1.3 \cdot 10^{-6}$	$6 \cdot 10^{-6}$
K_1 (simulation)	150	160	170	180
K_1 (analysis)	116	123	134	142

approach by transforming the two-resource allocation problem into two independent resource allocation problems. This allows us to explore the maximal benefits of both buffer sharing and bandwidth sharing. This is important, as the efficacy of buffer sharing and bandwidth sharing for sources with different time scales is quite different. As a consequence our loss probability estimate (25) that explores the "best" resource separation is expected to provide better results than the approach in [10]. This has been confirmed by our numerical investigations. As an example, we compare our results in Fig. 13 with the corresponding example of Fig. 13 in [10]. The parameters of the classes are swapped in their case, so our class 1 corresponds to class 2 in [10] and vice versa. In Fig. 15, we plot the admissible region obtained using both approaches, for loss probabilities of 10^{-9} and 10^{-5}. The bandwidth and buffer size are fixed to $C = 45$ Mb/s and $B = 1000$ cells, respectively. The admissible region computed by means of (25) is larger for all the loss probabilities considered.

Finally, we evaluate the accuracy of our approach by comparison with simulation. In Table II, we list the numbers of class-1 admissible sources K_1 obtained from simulation and our numerical approach for various loss probabilities, where $C = 45$ Mb/s and $B = 1000$ cells. The simulations results are taken from [10]. As expected, our approach provides a conservative estimate of the number of admissible sources when compared with simulation.

V. CONCLUSION

In this paper, we studied the problem of resource allocation and control for a network node with regulated traffic. Both guaranteed lossless service and statistical service with small loss probability were considered. We investigated the relationship between source characteristics and buffer/bandwidth tradeoff under both services.

For guaranteed lossless service, we identified the optimal resource allocation scheme for a network node with finite bandwidth and buffer space, and found that the optimal resource allocation scheme suggests an interesting time-scale separation of sources sharing the network node. With respect to this time-scale separation, the time scale of a source can be defined and the optimal buffer/bandwidth tradeoff determined by the sources' time scale. For statistical service with small loss probability, we presented a new approach for estimating loss probability in a shared buffer multiplexor with extremal ON–OFF periodic sources. Under this approach, the optimal resource allocation for statistical service is achieved by maximizing both the benefits of buffering sharing and bandwidth sharing. The optimal buffer/bandwidth tradeoff is again determined by the time-scale separation and reflects the efficacy of buffer sharing and bandwidth sharing among sources with different time scales. Through numerical investigations, we illustrated the relationship of source time scale and the optimal buffer/bandwidth tradeoff and discussed the implications of our results in resource allocation and call admission control.

Our results have many other implications in network design and control, such as network dimensioning and traffic shaping, in addition to resource allocation and call admission control in a network node. This will be the subject of future research.

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